

When was a dissonance a dissonance?

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Abstract

Definitions of dissonance have been in flux since the dawn of polyphony. What was never in question was that some combinations of tones were found to be more acceptable than others. We examine some of the main ideas of consonance and dissonance in European art music from roughly 1200 to 1600. The underlying questions are these: If notions of what dissonance kept changing, to what degree can the essence of dissonance be said to be perceptual? To what degree cultural? How does cognition intersect these? How are these questions pursued in historical and computational musicology? Do these fields complement or contradict one another?

Introduction

Historical and computational approaches to musicology seek to answer similar questions in different ways. Additionally, important differences of methodology occur within historical musicology according to the time, place, and repertory concerned. Scholarship of the tonal period is rich in detail, because editions of music are plentifully available. With respect to early polyphony, however, editions exist sporadically or not at all. Commentaries may be sparse because of the paucity of editions and a consequence lack of access to sources. More uncertainties make exist concerning the interpretation of both music and textual accounts of it.

In computational musicology, the much younger field lacking discourse on best practices, the detail missing in historical studies can sometimes be determined or inferred from the investigation of encoded sources, but until very recent times, little music from before 1600 had been encoded, and much remains unedited. Rules laid down by theorists can be verified or refuted—but only to the extent that source material is available.

Neither field adequately addresses questions of cognition. Our manner of experiencing and understanding music today is significantly different from the way it may have been in its own time. In particular, late medieval music may be perceived by novice listeners today as highly dissonant, but this view is colored by lack of familiarity with the idiom and with grammars of intervallic progression that may be foreign, if not precisely dissonant. Very little is understood about tuning systems before 1500, apart from the fact that actual tuning (as opposed to mathematical descriptions of the overtone series) varied from place to place.

Leach (2011: 58-59) points out that a significant number of twentieth-century composers who considered Machaut (c. 1300-1377) an inspiration to them, as editions of his music began to multiply in the middle years of the twentieth century. Schoenberg's effort to "emancipate the dissonance" as defined in tonal contexts located a parallel proving ground in pretonal music. Yet the idea resituated in the fourteenth century was clearly anachronistic. Leach's historiographical study of the reception of Machaut's poetry and music shows that over the past three centuries writers of diverse nationalities and perspectives have rarely escaped the fate of projecting onto this very early polyphonic repertory the intellectual stamp of their time or place. Nor is it likely the enquirers today will do better. There is a third basic aspect of coming to grip with dissonance in this one repertory (a substantial portion of which is now viewable online at <http://gallica.bnf.fr>), which is that over the evolution of Machaut editions in the past century the interpretation of early notation has often been challenged. Thus in listening to performances and recordings today it is important for those interested in details of content to bear in mind that interpretations continue to vary. The "tonal truth" is neither singular nor necessarily consistent.

These fundamental considerations of the music of Machaut, generally regarded as the greatest composer of his time, pertain to varying degrees to all preserved repertoires of early European polyphony. (1) We cannot really know what sense those living between the thirteenth and sixteenth centuries actually made of the music they heard. We cannot expect that there was any highly evolved sense of chronological "evolution" of music style or of rules governing the composition of music, even though countless scholars of the past century have picked through the minutiae of prescriptive treatises. (2) The idea of chronological compartmentalization was itself an artifact of the later seventeenth century, and the period during which a wide range of polyphonic practices was coalescing into what we now call tonality. (3) The details of modern editions of early music have been the frequent cause of sharp differences of opinion. Overwhelmingly, the cause of dissent is related in some way to dissonance. (4) In the making of medieval music (for which no intermediation by recorded example was available) aural ambience and cultural expectation emanated from worlds vastly different from our own—from monasteries isolated from one another, from courts where performers from throughout a region might be assembled, and from occasional events such as weddings and funerals of the lords of the land. Monastic and court audiences had well-honed tastes, while the opportunities of large peasant populations to hear music were necessarily limited to amateur music-making.

Concepts of consonance from 1000 CE to 1400 CE

2a. The Tetrachord

A fundamental requirement in understanding the index of consonances permissible in the later Middle Ages is a grounding in the underlying theory of monophonic music. By Machaut's time this framework was based on roughly two millennia of practice, since it rested largely on the teaching of

Greek tetrachordal systems. The supremacy of the tetrachord rested on the extensive music theory found in Greek writings of the age of Socrates, Plato, and Aristotle (Mathiesen 1999). Long passages describing the many varieties of tetrachords were still be reiterated (in Latin translation) in writings of the later Middle Ages.

Tetrachords offer a useful pivot between the modes of the late Middle Ages and the scalar systems of the Eastern Mediterranean, Middle East, and some traditional East Asian repertoires. This flexibility rests on the alternative methods of coupling two tetrachords. As long as they were separated by a whole tone, two *disjunct* tetrachords inevitably formed a diatonic scale. Two *conjunct* tetrachords formed a heptatonic scale. Such conjunctions have many analogues in recent monophonic non-Western music but seem to have disappeared from Western music in parallel with the formalization of theories of tonality in the eighteenth century.

Although a broader range of intervallic sequences within tetrachords is recognized today, the Greek systems presumed all intervals to be either a whole tone or a semitone. This restriction, and the durability of the concept into the later middle ages, must owe to the assumed relationship between the theory and the capabilities of harp-like instruments (lyres in their many descendants). Illustrations of “King David and his harp” persisted into the eighteenth century, but by way of extant examples we are mainly dependent on images of the *kithera* baked into the paintings on Greek *amphorae*. (The wind and percussion instruments of ancient Greece were largely ignored by medieval theorists. Plato held that the *aulos* inspired lascivious behavior. The prohibition of end-blown flutes persisted in Roman Catholic Church regulations through the seventeenth century.) We can never know precisely how these instruments were tuned. Elaborate mathematical prescriptions for tuning encourage us to think we know, but the ear of the musician who tunes the instrument will ultimately determine exactly what listeners hear.

The tenth-century monk Hucbald (Hucbaldus) devised a scheme for teaching novices how to correctly render liturgical chant on the lyre. In the absence of notational systems, Hucbald chose to explain chant melodies in relation to the strings of a six-string instrument that was available in his monastery. After a substantial reiteration of the Greek tetrachord system he provides the schematic diagram which can be viewed at e-Codices (the Swiss Virtual Library of Manuscripts, Hucbaldus, f. 119).

It is arguably the case that his effort led to the development of staff notation, for he used a graph of the instruments strings as a grid onto which he suggested mapping musical movement, although he may have been more nearly mapping pitch onto a harp tabature. Various systems of neumes intended for the voice only were otherwise employed in the conveyance of melodies.

2b. Theory vs Practice

As with all enquiries into the content of music, the understanding of late medieval notions of dissonance must be deduced bilaterally from two contradictory domains—the music that survives and the written prescriptions, largely theoretical, of those writers we now call music theorists. In reading theorists, we must accept that since most music written before the start of music printing (c. 1500) does not exist, and that therefore they may be extrapolating from repertoires to which we have no hope of access. Apart from the perennial gap between theory and practice, we can be fairly confident that some gap existed between the precepts of theorists and the perceptions of listeners. Recent research in ethnomusicology should cause us to question whether listeners (other than musicians themselves) would have noticed any of the phenomena mentioned here as dissonant. Ethnomusicology teaches us that musical cognition can depend more on social factors than auditory phenomena. List 1985 offers an extreme case. In the vertical social alignments of royal households and rich monasteries, listeners would have observed the taste of the king or bishop irrespective of musical content. Nonetheless the evolution of thinking about dissonance is instructive in documenting the ever lower threshold (in intervallic size) of tolerable tone combinations.

The most significant caveat about inferring cognition from available evidence for earlier centuries is that for most of the period in question theorists never discussed textures involving more than two voices, even though as composers they may have constructed music for three or four voices (Fuller 1986). This by itself creates an enormous gap between theory and practice.

2c. Early polyphonic consonance

In theoretical writings of the later middle ages we read not of dissonance but only of permissible consonances and their combinations. Unisons, fifths, and octaves were classified as “perfect” intervals, all others “imperfect. In Fuller’s discussion Machaut’s “sonority types” fall into three classes:

Sonority type	Definition	Examples
<i>Perfect</i>	Perfect + perfect	Unison + fifth
<i>Imperfect, Class 1</i>	Perfect + imperfect	Fifth + third; octave + sixth
<i>Imperfect, Class 2</i>	Imperfect + imperfect	Third + sixth

This basic framework, which permitted a total of nine combinations of two intervals, does not specifically address intervallic quality—whether thirds and sixths are major or minor, for example. It can be assumed that neither two major nor two minor imperfect intervals could not be used together; in quality they had to complement one another. Otherwise Machaut would have sounded like Debussy (through combinations of major thirds) or Mozart (through combinations of two minor thirds).

The modern perception of “hollow” combinations (e.g., imperfect, class 1 triads) indicates how what one age perceives as appropriate may be completely eclipsed by later interpretive conventions. From this phenomenon we deduce that *cultural expectation forcefully frames our interpretation of what we hear*.

2d. Organum

In the three-voice scheme shown above, which is derived from via Fuller’s 1986 account from theorists writing in the fourteenth century, the role of fourth is curiously mute, since in a two-voice texture the fourth had been viewed as “the most perfect of all intervals.” In modern tonal theory it is a perfect interval, since it is an inversion of a fifth and since its tuning relative to the prime is absolute. The fourth was fundamental to tetrachordal theory, since the outer tones of that sequence always formed a fourth. However, monophonic music constrained by tetrachords moved stepwise.

If we turn to the repertory of organum (the earliest kind of organized polyphony from Europe), we encounter contexts in which a second voice paralleled the pre-existing voice consistently by a fourth.

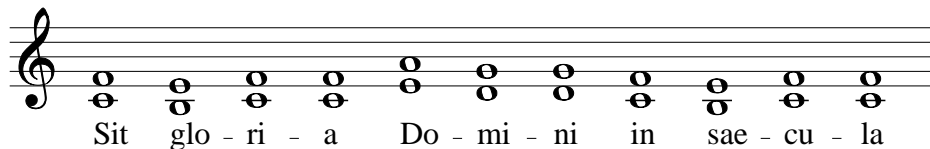


Figure 1: Ninth-century organum featuring parallel fourths. (Transcribed the recording made under the supervision of Gerald Abraham for the *History of Music in Sound, Vol. 2: Early Medieval Music up to 1300*, ed. Dom Anselm Hughes. Oxford University Press, 1953.

Organum (the term meant to “sing in symphony”) arose in the Frankish Empire (particularly in Franconia) by the end of the ninth century. Its uses within the liturgy were diverse and its types proliferated. Yet there was a substantial coherence within the repertory. It was always agreed that the preexisting melody would predominate, so that the listener would not be confused. In this sense the equality of voices sought in modern performances of early polyphonic music are misleading. At the same time, Leach (2011: 63) observes that the mid-twentieth century revival focus on instrumental doubling finds no support on prescriptions of Machaut’s time. We cannot really be sure by what means the melody was intended to stand out.

The “parallel” phase gave way to what came to be known as Aquitanian and free organum. In free organum the voices moved in synchrony but in sources such as the tenth-century Winchester Troper parts could cross but oblique movement was common. This eventually induced a need to

devise mechanisms for synchrony. The adoption of a series of rhythmic feet (borrowed from the poetry being set to music) led to the widespread use of isorhythm—a single, repetitive foot used in the principle voice throughout a musical setting. The trance-inducing results were widely adopted in secular works such as motets. (In motets of the later middle ages the text was freely composed and could be naturally conformant to a poetic meter.) In Aquitanian organum much energy was injected into the added voice by elaboration vowels in the text with vocal melismas. Notre Dame organum, coincident with the first century of operation (1163-1238) of the new Parisian cathedral, introduced a number of innovations, including sectional differences of style. Isorhythm formed the basis of elaborate principles of a musical architecture that could produce cunning constructions.

From a perceptual perspective there is reason to allow that listeners to isorhythmic works were far more entranced by the repetition and impressive by the polyphony, though melismatic usage would certainly have attracted attention to the florid voice.

2e. The rise of the hexachord

By the fifteenth century tetrachords had given way to hexachords. Hexachords were not combined end-to-end in conjunct or disjunct ways. Three basic types outlined below were permitted to overlap.

Hexachord genre	Notes required
<i>Natural</i>	C-D-E-F-G-A
<i>Soft</i> (molle)	F-G-A-Bb-C-D
<i>Hard</i> (dur)	G-A-B-C-D-E

One tone in each set of six could be modified (“inflected”) in order preserve a precise intervallic sequence: whole-whole-half-whole-whole. The natural hexachord could be *mutated* into the soft by adding a flatted seventh, which would then be reinterpreted as the fourth degree of the new hexachord.

As hexachords passed into modal practice and as the diatonic modes that resulted gradually coalesced into the simpler configuration of diatonic scales, hexachordal guidelines hardened into rules which had the effect to evolving in ways that become recognizable from later practice. If, for example, a B extending the range of a natural hexachord beyond the statutory sixth was flatted, the transgression came to be seen as an elastic addition. The out-of-bounds pitch might be penalized for overshooting a boundary by being forced to “turn around” and return to its starting place. (In later tonal theory a flatted note was not permitted to ascend, whereas a sharpened note was not permitted to descend.)

The shift to hexachords precipitated an avalanche of new thinking. Guido d’Arezzo (991/2-1050), a Benedictine monk in central Italy, is noted for his efforts to simplify music instruction through the

use of solmization (the assignment of syllables to pitches of the hexachord), the adoption of clef signs, and the invention of what later became known as the Guidonian Hand, a mnemonic device assigning solmization syllables to each finger joint in order to enable singers to cope with the mutation of hexachords. Guido's "hand" first appeared in his *Micrologus*, a manual of c. 1030, originally intended to train singers for the cathedral of Arezzo. It covered both monophony and polyphony.

Guido objected to the use of parallel octaves and fifths in organum. He preferred "softer" effects. ("Soft" became a fixed designation of the F hexachord, which took Bb instead of B, since the melodic fourth ("the most perfect of all intervals") needed to be preserved. Guido described oblique motion and developed rules for it. He preferred voices to converge at the end of a piece by stepwise motion. His views promoted subtlety of expression through enabling greater tonal control.

The Rise of Dissonance: 1400-1750

The great legacy of the hexachordal system was the varying interpretations that arose about the "mutation" of hexachords in polyphony. The performance of unnotated alterations of pitch, in conformance with the dictates of the hexachordal system, constituted the practice of *musica ficta*—"falsifying the music" in order to satisfy the rule that certain combinations of sounds (especially tritones) were to be avoided at all costs. Instances in which a best solution in one voice leads to an awkward set of choices in another are common. Professional groups today prohibit the triton (an augmented fourth or diminished fifth)—the *diabolus in musica*—no matter how it is approached or followed.

The fluidity that attends the practice leaves researchers working from modern editions vulnerable to mischaracterizing harmonic practice. In some works (and modes) the difference between altered and directly reproduced pitches can be negligible, but others, such as the Josquin motet called "Absalon, fili mi," offer wide scope for variance. In this work (occupying only three pages in print: see <http://jrp.ccarh.org/cgi-bin/jrp?a=notation&f=Jos1401>) 41 tacit alterations occur.

"Absalon, fili mi" contributes to a continuing discussion of helical schemes of tonal relations in music of the fifteenth and sixteenth centuries. For an earlier generation of musical scholars, these cycles contributed to the perception of growing use of chromaticism. Akin to the "Shepard tone" (an auditory illusion), some Renaissance composers were savvy to this effect. An apparent downward spiral that never falls below the gamut, no matter how many cycles it iterates, occurs in "Absalon, fili mi." While there was no predominant Circle of Fifths, pieces employing cascading series of fourths and thirds can be found. Edward Lowinsky (1946) was one of the mid-century

musicologists who invested the phenomenon with great powers. He found them helical relations both “self-generating” and “eloquent.”¹

In her dissection of Lowinsky’s arguments on chromaticism, Bonnie Gordon notes that Lowinsky’s understanding of key and pitch were entirely modern, a condition that is undoubtedly *a propos* of many of other studies. In particular, she observes that “Lowinsky’s sense of what constitutes a chromatic tone is rooted in hearing music vertically; for him, chromaticism function[s] structurally, not decoratively. ...Scholars assume that sixteenth-century musicians understood chromaticism as primarily melodic; polyphonic textures became chromatic as a result of melodic inflections, and pieces that sound highly chromatic to our ears may not have sounded so to theirs.”² In support of this view, she marshals evidence from recent writings by James Haar (1977) and others that diminished fourths, fifths, and octaves were more common than scholars of Lowinsky’s generation supposed.

Cognitive researchers should note that, however, that Gordon’s vertical-horizontal dichotomy touches an area of ongoing controversy. How did performers and listeners perceive tonal relationships as they evolved over the course of a performance? Did their attention shift between orientations? From tetrachords discussions onward, tangential reasons for supposing that thinking about harmonic relations fluctuated from moment to moment between simultaneities and note sequences within voices accumulate. Yet proof is elusive.

Definitions of consonance have always addressed simultaneities. Definitions of prohibited relationships (loosely “dissonances”) were more variable. Contexts were often not exhaustively described. If Lowinsky erred in bringing a “Bach chorale” perspective that was too strictly vertical, some would argue that an entirely linear perspective is equally unlikely to have prevailed. Did musicians shift their attention between vertical and horizontal tone combinations as a work progressed? If so, they would surely have listened for consonance on final tone combinations and strongly accented beats. Were rules meant only for conspicuous moments of coincidence between parts?

Fuller notes (1986: 43f) that the question of *why* consonance was required at terminating points was addressed by the anonymous author of “Cum notum sit,” who writes that “If the song were to end on an imperfect consonance, then the [listener’s] mind would remain suspended, and would

¹ Lowinsky’s writings on harmony provoked strong reactions, which increased over time. In his Bloch lectures at Berkeley (1956) he made the case that tonality developed earliest in dance music of the Renaissance. Carl Dahlhaus (1990), who was little known at the time, was among those who heaped scorn on Lowinsky’s work (1990).

² Gordon, pp. 335f.

not find repose...until it heard a perfect sound.”³ An English theorist of the same time states that the unison is perfect “because of its immobility” (1986: 44). Imperfect intervals are so named because of their *instability*.

Confronting the Music

The Music via Notation

We might suppose that the treatment of perfect and imperfect interval combinations, the avoidance of parallel motion,⁴ the convergence of voices at a cadence and so forth would be easily verifiable in the music itself. However, it has only become practical in recent months to undertake broad-brush evaluations of these kinds in substantial quantities of early polyphony. At Stanford University we are fortunate to have under development the Josquin Research Project (<http://jrp.ccarh.org>), which at this writing contains 550 works (or work sections) by Josquin and his contemporaries, with *musica-ficta* alterations realized by project director Jesse Rodin. These resources are well suited to the evaluation of observance of rules.

Figure 2 shows the relative frequencies of harmonic intervals in polyphonic music for three repertoires: nearly complete extant opuses of early Renaissance composers Johannes Ockeghem (1410/25-1497) and Josquin des Prez (c1450-1521), as well as 370 four-part chorales by J.S. Bach used as a reference. Musically interesting information which can be read from the plot include:

- The perfect 5th is the most common harmonic interval in all repertoires.
- The minor 6th is the least common consonant interval (excluding unison).
- P5/m3/M3 are more common than their inversions, which occur in the same ranked order P4/M6/m6 (with the exception of M6 being slightly greater for Ockeghem than P4).
- All perfect/imperfect consonances are more common than dissonances.
- Bach chorales de-emphasize octave/unison intervals.
- Bach chorales increase frequency of P4/M6/m6 intervals.
- Bach chorales double the use of tritone (aug4/dim5).

³ As for the status of the (imperfect) third and sixth, another unnamed writer of the fourteenth century (Fuller, 1986: 44) observed that such intervals “produce imperfect consonance because they are inclined to ascend or descend to ... perfect intervals...”

⁴ <http://josquin.ccarh.org/data?repertory=parallel>

- Bach chorales use more M2/m7 dissonances, but about the same for m2/M7.

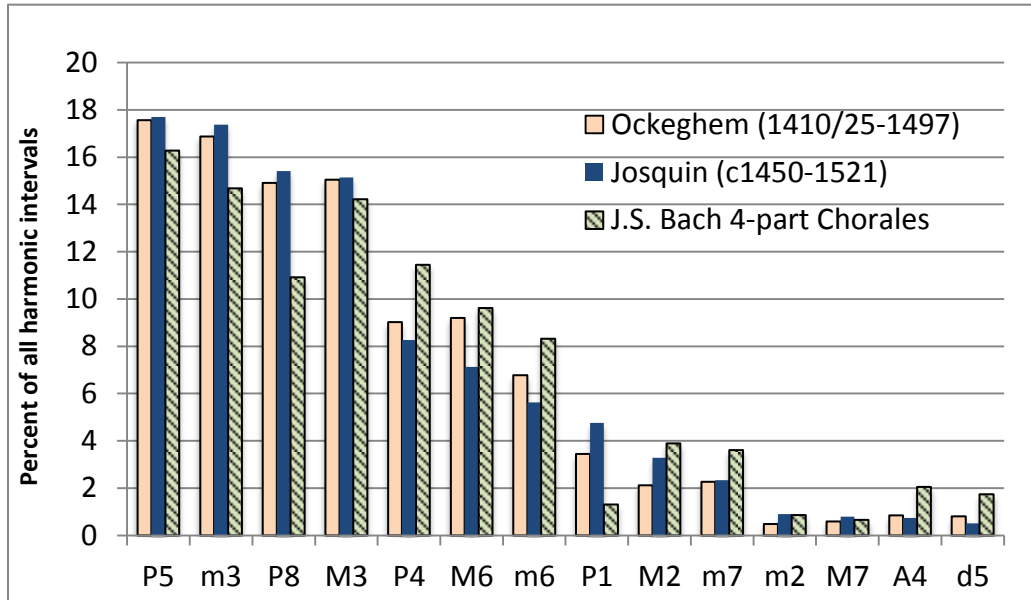


Figure 2: Frequency of harmonic intervals in three repertoires of polyphonic music. Intervals larger than an octave are transposed down. Total interval counts for each repertoire: 149,205 (Ockeghem), 326,829 (Josquin), 180,374 (Bach chorales).

The general harmonic-interval trend between early Renaissance repertoires and Bach chorales is a 30–40% increase in dissonant intervals (other than m2/M7) which corresponds to a 10% relative decrease in perfect consonant intervals:

	Ockeghem	Josquin	Bach chorales
Perfect Consonances	44.5%	46.1%	40.0%
Imperfect Cons.	47.9	45.3	46.8
Dissonances	7.6	8.6	13.2

In terms of triadic sonorities summarized in Figure 3, Ockeghem gives a nearly equal weighting to major and minor triadic sonorities in all inversions. Josquin increases use of the root form of these sonorities as well as uses minor triadic sonorities more often than major. All repertoires prefer diminished triadic sonorities to be in first inversion, while major/minor sonorities are most common in root position, followed by first inversion, and second inversion being least common.

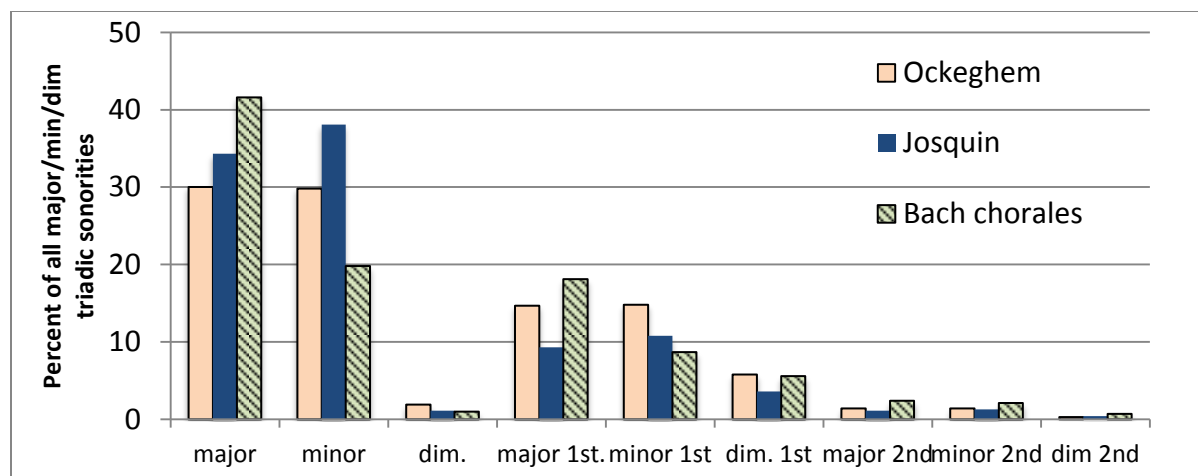


Figure 3: Relative amounts of triadic sonorities in three repertoires, separated into root, 1st and 2nd inversions.

The Music via Performed Sound

At a more subtle level of enquiry, the implications of Gordon’s questions about aural interpretation of mode (and thus of consonance) find a fertile and illuminating discussion in Margaret Bent’s classic “Diatonic Ficta” (1984), where the artificiality of equating hexachordal positions with exact pitches (as measured in frequency or cents) is discussed at length. Such terms are “chromatic” and “accidental” interject modern editorial judgment that may be misplaced in the context of Renaissance vocal polyphony, where performers were trusted to make their own *ad hoc* determinations, with a latitude that sometimes have defied the rules-of-thumb generally employed today. In particular, diatonicism was found in melody (that is, in horizontal relations). Discussions based on a vertical orientation do not pertain. This enquiry is extended and refined in Bent (1996), where she reaffirms the view that the relationship between written and sounding pitch varied, particularly for music before 1500.

A final caveat for researchers trying to make sense of notions of consonance and dissonance in early polyphony arises from the extensive work of Collins Judd (1992 et al.). Exploring question earlier raised by Harold Powers (1981), she maintains that a perceived dichotomy between Renaissance modes and “tonal types” did not exist. She points out (1992: 430) that rules for the determination of mode were developed in the first instance for monophonic music. The defining points were the final tone, the species, and the ambitus. Clef was later added in Pietro Aron’s *Trattato della natura et cognition di tutti gli tuoni* (1525), where it was again music by Josquin (here motets) that formed the basis for application of the earlier precept. Collins Judd offers her own principles as modern ones (1992: 464) but insists they are founded on the practicalities of absorbing (as Renaissance musicians had to) the roots of a tradition based on plainchant with “hexachordal manipulation of the gamut” and with contrapuntal principles of the time.

Discussion

The concept of dissonance seems always to have been fluid. Sharp pronouncements by theorists and scholars notwithstanding, definitions were in continual flux. On the level of generalization, we can ask whether if notions of dissonance kept changing, the essence of dissonance be said to be have been perceptual. It seems closer to the truth to claim that the notion that some tonal relationships were more appropriate than others was a long-lived idea, but that one can only fix such relationships precisely in relation to temporal and local contexts. Durable notions associated with static content are elusive. Wegman (1997) reminds us that listening was a community activity in early modern Europe. Bent (1984, 1996) and Collins Judd (1992) demonstrate ways in which a music-theoretic orientation favoring rule systems must be tempered by an appreciation of the degree to which practitioners of early polyphony determined their own tonal content and the ways in which apologists (i.e. Renaissance theorists) conflated principles of monophony with those of polyphony. Fuller (2011: 79) suggests that early theorists such as Marchetto di Padua allowed that singers might “feign” to elicit from listeners particular tonal expectations, though her interpretation differs from one by Leach in asserting that Marchetto spoke of certain contrapuntal contexts rather than specific the chromatic semitone *per se*.

Disagreements over the interpretation of what early theorists said are as common as those over the editing of accidentals (*musica ficta*), of course, and although a systematic evaluation of successive vertical sonorities can tell us much about the content and grammar of the music, they cannot really tell us how cognition intersected these complex phenomena. We can look forward to expanding opportunities for systematic verification of usage as documented by notation, though a thorough understanding of early polyphony will still require a significant increase in the number of available editions. For questions that depend on sounding pitch we shall never be able fully to address eras predating the advent of recording technologies. For now, dissonance remains stubbornly in the ear (and mind) of the beholder.

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