Base-40 arithmetic for music apps

MUSIC 253/CS 275A STANFORD UNIVERSITY

What problem does base-40 arithmetic solve?

Preservation of enharmonic spelling

Uses:

- Analysis
- Interval invariant transposition
- Make "dumb" representations (e.g. MIDI) smarter

Where did the base-40 concept originate?

Conceived by Walter Hewlett (1986); first pub 1992

Goals: enharmonic spelling preservation, correct analysis, correct transposition

Reproduced at http://www.ccarh.org/publications/reprints/

Further elaborated in U.S. Patent 5,675,100 (7 October 1997)

http://www.google.com/patents/US5675100

Subdivisions of the octave

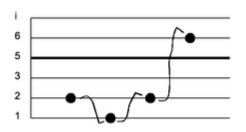
and their calculations

Based on name-classes (i.e. pitch names): diatonic

Name-classes extended to chromaticism

- Include single sharps and flats
 - Exclude E/F and B/C
 - Include E/f and B/C spans
- Accommodate alternative tuning
 - Follow equal temperament
 - Follow another tuning system





Common bases in musical arithmetic

Subdivisions of the octave

- Base 7 (diatonic)
- Base 12 (semi-chromatic; MIDI)—favors equal-tempered sound
- Base 21 (fully chromatic through 1 #/b)—favors simple notation
- •Multiples of 7 (19, 35....)
- Base 40 (fully chromatic through 2 #/b)—favors tonal analysis, transposition, accurate notation

Why Base-40? Arithmetic complements

Musical literacy

Tonal legibility (common-practice era)

Musical computation in integer arithmetic

Music: Intervallic complementarity

Base-10 complementarity:

If interval = 3, complement = 7

If interval = 6, complement = 4

Interval sizes and qualities

Interval classes:

M = major

M = minor

P = perfect

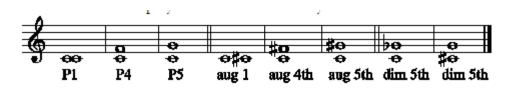
Aug = augmented

Dim = diminished









Interval classes

Rest on **number of semitones** between two pitches &

The interval class (related to overtone series)

- Prime, 4th, 5th, 8ve = "perfect" intervals
- ∘ 2nd, 3rd, 6th, 7th = imperfect intervals

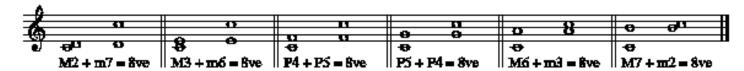






Intervallic complementarity: intervals

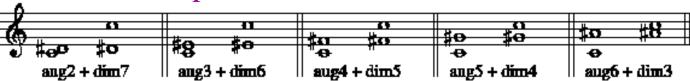
The complement of an interval is the one required to complete the 8ve



If M2, then m7 = complement etc.



If aug2, then dim7 = complement

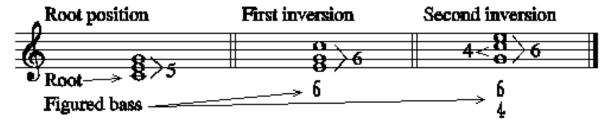


Intervallic complementarity: chords

Triads (3-note chords) consist of two interior intervals and an outer interval



"Position" of chord describes arrangement of intervals



Integer arithmetic in digital analysis

Binomial solutions: Brinkman, Böker-Heil

Required 3 params (pitch name, octave number, inflection)

Arbitrary mappings: C=10, D=20, E=30....

 Same-sized intervals do not always produce same numbers (depends on endpoints: F-E = 10, Eb-D = 9)

Hewlett's base-40 system is interval-invariant:

- it produces consistent arithmetical results
- irrespective of endpoints and without binomials
- Preserves complementarity customary in music theory

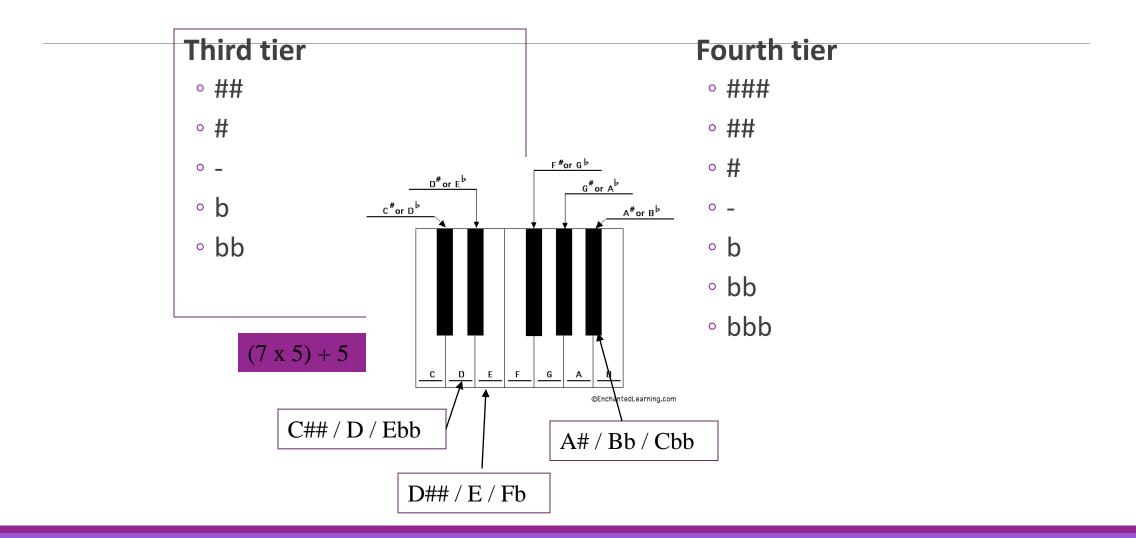
Enharmonic-notation tiers

Physical instrument ABCDEFG F #or G ♭ D[#] or E G[#]or A[♭] c[#]or D[♭] A[#]or B[♭]

Cultural apparatus

- Letter names
 - Base-7 (0 #s/bs)
- Octave numbers
 - Base-12 (1#/b)
- Inflection names
 - Base-21 (1#/b)
- Inflection names
 - Base-40 (2#/b)

Enharmonic-notation tiers



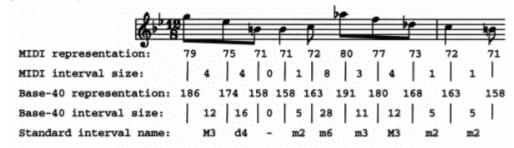
Representation		11	Computation of Intervals							
201	_	1								
	-		11							
22	-	2	H							
	-	3	П							
1	_		l	-						
	-	-		Inter	val	Delta	Interval	Delte		
141	-	5	П	perfec	t unison		perfect octave	40		
	-	6	11		mison	ĭ	dim. octave	39		
2	-	7	Н							
1		_	И	din. e	econd	. 4	aug. seventh	36		
1	-	8	Н		second	5	major seventh	35		
	-	9	П		second	6	minor seventh	34		
1		10	Ш	aug. s	econd	7	dim. seventh	33		
1			11							
47	-	11	П	dim. t		1.0	aug. mixth	30		
	-	12	11	minor		11	major sixth	29		
b 1	-	13	П	major		12	minor eigth	28		
	_		11	aug. t	hird	13	dim. sixth	27		
X.	•	14	П							
	-	1.5	11	dim. f		16	aug. fifth	24		
1	-	16	1.1		t fourth	17	perfect fifth			
ā	_		11	aug, f	ourth	18	dim-fifth	22		
#1	•	1.7								
71.	-	13	-		***************************************					
_	_									
_	=	19								
1	_	19 20	2.			f a simple	interval is forty :	ninue		
1	_	19	2.		version o	f a simple	interval is forty	sinue		
1	_	19 20		that i	nterval.			minus		
	_	19 20 21	2.	Interv	nterval.	e computed	across the B - C	minue		
h	_	19 20 21 22 23		Interv	nterval.	e computed		ninue		
1	_	19 20 21 22		Intervoctave	rals may be boundary	e computed without e	across the B - C			
1	_	19 20 21 22 23	3.	Intervoctave Compou	rals may be boundary and interv	e computed without s	across the B - C	1		
1 1 1 1 1	_	19 20 21 22 23 24	3.	Intervoctave Composito int	rals may be boundary and interversels by	e computed without s	i across the B - C extra calculations. Is tenths are related brence of an octave	1		
1 1 1 1 1 1 1	:	19 20 21 22 23 24 25 26	3.	Intervoctave Compout to int	rals may be boundary and intervene by tenth i	e computed without a als such a the diffe s 12 + 40	across the B - C extra calculations. Is tenths are related extrance of an octave = 52.	s (40).		
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	_	19 20 21 22 23 24 25	3.	Intervoctave Compouto inta majo	vals may be boundary and intervented by tenth intervented by the tions: I	e computed without a als such a the diffe s 12 + 40 intervals i	across the B - C extra calculations. Is tenths are related rence of an octave = 52.	1 (40). ide the		
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	:	19 20 21 22 23 24 25 26 27	3.	Intervoctave Compout to int A majo Limite set, e	rels may be boundary and interversels by ar tenth interversels. It is a second to the	e computed without a als such a the diffe s 12 + 40 intervals i three or m	i across the B - C extra calculations. Is tenths are related greenes of an octave = 52. Involving notes outs: lore sharps or flats.	1 (40). ide the		
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	:	19 20 21 22 23 24 25 26 27	3.	Intervoctave Compout to int A majo Limite set, e cannot	rels may be boundary and intervels by ar tenth intions: I it is to be computed by the becomputed by th	e computed without e als such a the diffe s 12 + 40 ntervals i three or m ted proper	across the B - C extra calculations. Is tenths are related exerces of an octave = 52. Involving notes outs: lay from this repre-	1 (40). ide the		
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	:	19 20 21 22 23 24 25 26 27 28 29	3.	Intervoctave Compout to int A majo Limite set, e cannot sentat	rals may be boundary and intervervals by or tenth i stions: I to be computed by some state of the state of th	e computed without s als such a the diffe s 12 + 40 intervals i three or a ted proper e unusual	across the B - C extra calculations. Is tenths are related rence of an octave = 52. Involving notes outs: ore sharps or flats, ly from this repre- intervals will have	1 (40). ide the		
1 1 1 1 1 1 1	:	19 20 21 22 23 24 25 26 27	3.	Intervoctave Compouto int A majo Limite set, e cannot sentat Which	rais may be boundary and intervented by or tenth intions: I be computed by or tenth intions: I be computed by or tenth intions overlap to	e computed without e als such a the diffe s 12 + 40 intervals i three or m ted proper e unusual he nusual	i across the B - C extra calculations. Is tenths are related rence of an octave = 52. Involving notes outs: lore sharps or flats, ly from this repre- intervals will have for the standard	1 (40). ide the		
1 1 1 1 1 1 1	:	19 20 21 22 23 24 25 26 27 28 29	3.	Intervoctave Compouto int A majo Limite set, e cannot sentat Which	rais may be boundary and intervented by or tenth intions: I be computed by or tenth intions: I be computed by or tenth intions overlap to	e computed without e als such a the diffe s 12 + 40 intervals i three or m ted proper e unusual he nusual	across the B - C extra calculations. Is tenths are related rence of an octave = 52. Involving notes outs: ore sharps or flats, ly from this repre- intervals will have	1 (40). ide the		
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		19 20 21 22 23 24 25 26 27 28 29 30	3.	Intervoctave Compout to int A majo Limite set, e cannot sentat which interv	rais may be boundary and intervented by tenth intervented by the computations: If the computation of the com	e computed without a als such a the diffs s 12 + 40 intervals i three or m ted proper e unusual he numbers above. 2	i across the B - C extra calculations. Is tenths are related rence of an octave = 52. Involving notes outs: lore sharps or flats, ly from this repre- intervals will have for the standard	1 (40). ide the		
1 1 1 1 1 1 1 1 1 1		19 20 21 22 23 24 25 26 27 28 29 30	3.	Intervoctave Compouto int A majo Limite set, a cannot sentat which interv quadru	rais may be boundary and intervervels by r tenth i tions: I to compute the computer overlap rais given uple augme	e computed without e als such a the diffe s 12 + 40 intervals i three or m ted proper e unusual he numbers above. 3 inted unisc	across the B - C extra calculations. Is tenths are related rence of an octave (= 52. Involving notes outs: love sharps or flats, ly from this repre- intervals will have for the standard for example, the on between Cb1 and	d (40). ide the		
1 1 1 1 1 1 1 1 1		19 20 21 22 23 24 25 26 27 28 29 30	3.	Intervoctave Compout to int A majo Limite set, a cannot sentat which interv quadru c*11 h	rais may be boundary and intervented by the tervels by the tenth intervented by the tenth interv	e computed without e als such a the diffe s 12 + 40 intervals i three or m ted proper e unusual he numbers anted units and meted units and met	across the B - C extra calculations. Is tenths are related rence of an octave = 52. Involving notes outs: ore sharps or flats, ly from this repre- intervals will have for the standard or example, the m between Chil and m of 6, which is als	d (40). ide the		
1 1 1 1 1 1 1 1 1 1 1 1		19 20 21 22 23 24 25 26 27 28 29 30 31 32 33	3.	Intervoctave Compout to int A majo Limita set, a cannot sentat which interv quadru chil h the nu	rais may be boundary and intervals by or tenth intions: I i.g. with the computation. Some overlap trais given uple augments as an intember for	e computed without e als such a the diffe s 12 + 40 ntervals i three or m ted proper e unusual he numbers above. ? mbove. ? arvel valu a diminish	across the B - C extra calculations. Is tenths are related rence of an octave (= 52. Involving notes outs: love sharps or flats, ly from this repre- intervals will have for the standard for example, the on between Cb1 and	1 (40). ide the number		
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		19 20 21 22 23 24 25 26 27 28 30 31 32 33	3.	Intervoctave Compout to inta A majo Limita set, a cannot sentat which interv quadru c#1 h the mu limita	rais may be boundary and intervals by or tenth intions: I i.g. with the computation. Some overlap trais given uple augments as an intember for	e computed without s als such a the diffe s 12 + 40 intervals i three or m ted proper e unusual he numbers above. I inted unisc srval valu a diminish be remove	i across the B - C extra calculations. Is tenths are related reence of an octave in = 52. Involving notes outs: lore sharps or flats, ly from this repre- intervals will have in for the standard for example, the on between CP1 and a of 6, which is als led second. These	1 (40). ide the number		
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35	3.	Intervoctave Compout to inta A majo Limita set, a cannot sentat which interv quadru c#1 h the mu limita	rais may be boundary and intervented by r tenth i tions: I to compute overlap trais given uple augme as an intervention can	e computed without s als such a the diffe s 12 + 40 intervals i three or m ted proper e unusual he numbers above. I inted unisc srval valu a diminish be remove	i across the B - C extra calculations. Is tenths are related reence of an octave in = 52. Involving notes outs: lore sharps or flats, ly from this repre- intervals will have in for the standard for example, the on between CP1 and a of 6, which is als led second. These	1 (40). ide the number		
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		19 20 21 22 23 24 25 26 27 28 30 31 32 33	3.	Intervoctave Compout to inta A majo Limita set, a cannot sentat which interv quadru c#1 h the mu limita	rais may be boundary and intervented by r tenth i tions: I to compute overlap trais given uple augme as an intervention can	e computed without s als such a the diffe s 12 + 40 intervals i three or m ted proper e unusual he numbers above. I inted unisc srval valu a diminish be remove	i across the B - C extra calculations. Is tenths are related reence of an octave in = 52. Involving notes outs: lore sharps or flats, ly from this repre- intervals will have in for the standard for example, the on between CP1 and a of 6, which is als led second. These	1 (40). ide the number		
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36	3.	Intervoctave Compout to inta A majo Limita set, a cannot sentat which interv quadru c#1 h the mu limita	rais may be boundary and intervented by r tenth i tions: I to compute overlap trais given uple augme as an intervention can	e computed without s als such a the diffe s 12 + 40 intervals i three or m ted proper e unusual he numbers above. I inted unisc srval valu a diminish be remove	i across the B - C extra calculations. Is tenths are related reence of an octave in = 52. Involving notes outs: lore sharps or flats, ly from this repre- intervals will have in for the standard for example, the on between CP1 and a of 6, which is als led second. These	1 (40). ide the number		
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37	3.	Intervoctave Compout to inta A majo Limita set, a cannot sentat which interv quadru c#1 h the mu limita	rais may be boundary and intervented by r tenth i tions: I to compute overlap trais given uple augme as an intervention can	e computed without s als such a the diffe s 12 + 40 intervals i three or m ted proper e unusual he numbers above. I inted unisc srval valu a diminish be remove	i across the B - C extra calculations. Is tenths are related reence of an octave in = 52. Involving notes outs: lore sharps or flats, ly from this repre- intervals will have in for the standard for example, the on between CP1 and a of 6, which is als led second. These	1 (40). ide the number		
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36	3.	Intervoctave Compout to inta A majo Limita set, a cannot sentat which interv quadru c#1 h the mu limita	rais may be boundary and intervented by r tenth i tions: I to compute overlap trais given uple augme as an intervention can	e computed without s als such a the diffe s 12 + 40 intervals i three or m ted proper e unusual he numbers above. I inted unisc srval valu a diminish be remove	i across the B - C extra calculations. Is tenths are related reence of an octave in = 52. Involving notes outs: lore sharps or flats, ly from this repre- intervals will have in for the standard for example, the on between CP1 and a of 6, which is als led second. These	1 (40). ide the number		
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37	3.	Intervoctave Compout to inta A majo Limita set, a cannot sentat which interv quadru c#1 h the mu limita	rais may be boundary and intervented by r tenth i tions: I to compute overlap trais given uple augme as an intervention can	e computed without s als such a the diffe s 12 + 40 intervals i three or m ted proper e unusual he numbers above. I inted unisc srval valu a diminish be remove	i across the B - C extra calculations. Is tenths are related reence of an octave in = 52. Involving notes outs: lore sharps or flats, ly from this repre- intervals will have in for the standard for example, the on between CP1 and a of 6, which is als led second. These	1 (40). ide the number		
かれた は カスト は カスト は かない 大き		19 20 21 22 23 24 25 26 27 28 39 30 31 32 33 34 35 36 37 38	3.	Intervoctave Compout to inta A majo Limita set, a cannot sentat which interv quadru c#1 h the mu limita	rais may be boundary and intervented by r tenth i tions: I to compute overlap trais given uple augme as an intervention can	e computed without s als such a the diffe s 12 + 40 intervals i three or m ted proper e unusual he numbers above. I inted unisc srval valu a diminish be remove	i across the B - C extra calculations. Is tenths are related reence of an octave in = 52. Involving notes outs: lore sharps or flats, ly from this repre- intervals will have in for the standard for example, the on between CP1 and a of 6, which is als led second. These	1 (40). ide the number		

Base-40 Rules

Simple rule: Where a whole step exists between two key names, Insert a null token.



Example 4: "Seufzer, Tränen, Kummer, Not" from Cantata 21, Ich hatte viel Bekümmer



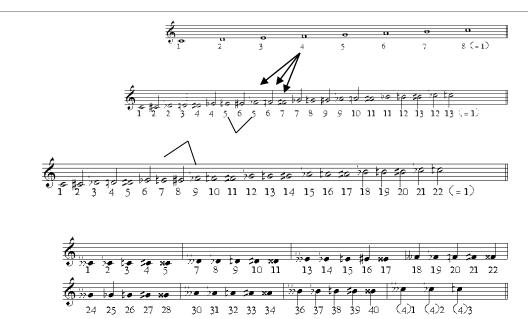
From Base-40 to enharmonic preservation

MIDI to base-7

MIDI to base-12

MIDI to base-21

MIDI to base-40



Solution: Translate from symbolic code to MIDIPlus

What is MIDIPlus?

In MIDI file format, a binary implementation of base-40

Replaces last 3 bits of velocity byte

Used to interpret key number

MIDIPLUS Correlation of Pitch Spelling to Specific MIDI Velocity Values

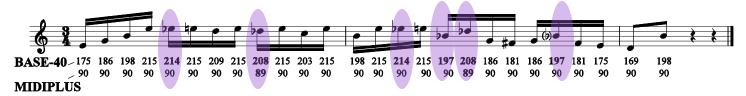
Value	Notated Pitch											
89	Dpp	Dþ	Epp	Fbb	Fb	Gbb	Gþ	Abb	Αþ	Врр	Cpp	Cþ
90	C	C#	D	Еþ	E	F	F#	G	G#	A	ВЬ	В
91	В#	B##	C##	D#	D##	E#	E##	F##	F###	G##	A #	A##

MIDIPlus in Printing

Raw MIDI to Notation (Bach Prelude in E Minor, BWV 855)

BWV855 RawMIDI

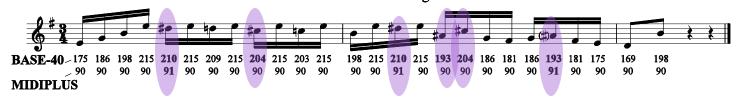
J. S. Bach WTC-I Fugue 10



Translation from symbolic code (MuseData) to MIDIPlus to notation

BWV855 With Correct Spellings

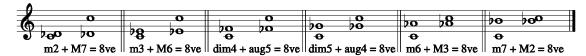
J. S. Bach WTC-I Fugue 10

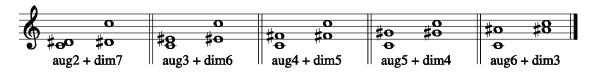


CS 275A/Mus 253 2023 ELEANOR SELFRIDGE-FIELD

Chords (intervallic complementarity)

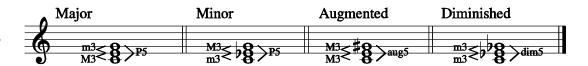
Intervallic complementarity







Chord definitions



Relevant handouts

Two translations of BWV 855 expressed with base-40

- E-Minor Fugue with enharmonically correct notation
 - http://esf.ccarh.org/MusicTheory Tutorials/Base40 Handout sup p1.PDF
- E-Minor Fugue **via MIDI**-to-notation:
- http://esf.ccarh.org/MusicTheory Tutorials/Base40 Handout sup p2.PDF

Music theory tutorial:

http://esf.ccarh.org/MusicTheory Tutorials/MusicTheory Computer Apps.htm

Remember Einstein!

