Base-40 Arithmetic for Notation-Based Applications

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Abstract

Almost twenty years ago Walter Hewlett focused on the numerical base 40 to facilitate several tasks in the manipulation of musical data. When the numeral 40 is taken as the measure of an octave, it is possible to prototype quickly any number of procedures including accurate transposition, accurate representation of enharmonic spellings, and interval-invariant complementarity in tonal music theory.

The obstacle to the use of the base-40 representation of pitch inheres in MIDI's lower-resolution base-12 description of the octave in relation to physical keys on an electronic keyboard. Through bit re-assignment, it is possible to greatly improve the "tonal legibility" of MIDI data created from an enharmonically sensitive symbolic code for music notation. Base-40 use improves on all known algorithms for interpreting raw MIDI signals in software. Base-40 capabilities are embedded in MIDI+.

Theoretical Background

In tonal music theory, every pitch requires three specifiers: a pitch name, an octave number, and a pitch inflection (sharp (or double sharp), flat (or double flat), natural). When limited to pitch name and octave number, one can describe only the *diatonic* (seven-tone) scale. A base-12 description can accommodate chromatic notes by number but not be enharmonic name (e.g., D#/Eb must share a number). Approximately 400 years of thinking about tonal music insist on distinguishing between enharmonic names for such "common notes" because, prior to the advent of equal temperament they truly were slightly different tones. The rational of tonal music theory is found on the recognition of this distinction.

Intervallic *complementarity* refers to a scheme of relationships in which particular sets of intervals, when paired, have a common tone which lies between two pitches which form an octave or one of its multiples. Because it is dependent on the acceptance of enharmonic differences of spelling, the nomenclature of intervallic complementarity, which lies at the heart of tonal music theory, cannot be adequately expressed in any base-12 system of nomenclature.

In computer applications, however, baase-12 descriptions of pitch abound. Some of the most prevalent base-12 systems include MIDI key numbers and pitch-class set-theory, which is widely used for the analysis of post-tonal music. The essentials of this vocabulary of working concepts is explained in the introduction of this talk.

Base-40 Arithmetic and MIDIPlus

The long-term use of MIDI data, the note-number specification of which was originally intended only as a hardware communication protocol, for higher-level software applications in musical notation and analysis has led to many compromises in the quality of the data representation within the resulting applications. Users have grown accustomed to the misinterpretation of enharmonic pitches, since the interpretation of MIDI note numbers for black notes of the piano tends to favor sharp "spellings" rather than flat one for purely arbitrary reasons. Over time, some compensations have accrued in the more robust programs for music notation. Nonetheless, the accuracy of pitch notation for works in minor keys, works with more than three or four sharps and flats in the key signature, works rich in enharmonic spellings, in "round-trip" or multiple transpositions of the same material, and in other procedures that require a secure basis for unambiguous pitch interpretation, MIDI-data remains ambiguous and therefore limiting.

Hewlett's base-40 representation of pitch, which has been in use at the Center for Computer Assisted Research in the Humanities at Stanford University (CCARH) since 1986, facilitates the use of simple integer arithmetic in transformational programs. It assigns a discrete integer to every pitch/pitch-inflection/octave to the five possible spellings for the seven note-names (A, B, C, D, E, F, G) within an octave through double sharps and double flats (##, #, -, b, b). Five null tokens are interleaved with these 35 values to create a total of 40 positions.

While there are many numerical representation schemes for chromatic and enharmonic pitch that lie between base-12 and base-40, we have continuously found that base-40 is the "lowest common denominator" for the reliable and unambiguous treatment of a full roster of pitch inflections as found in standard classical repertories of the nineteenth and earlier centuries. The system is, by the way, scalable for a series of larger bases that could accommodate music with triple or quadruple sharps and flats, but the need for such elaboration is extremely rare.

Since we create MIDI files from our in-house format called *MuseData*, we developed a system some years for mapping as base-40 representation onto the base-12 number line of MIDI. In this way, the enharmonic spellings which are encoded explicitly in our ASCII files are retained in MIDI. The advantage of this is that the same MIDI files can be interpreted more accurately than standard MIDI files (SMF) in notation applications.

This scheme formed part of the *MIDIPlus* format that was granted f U. S. patent No. 5,675,100 (October 7, 1997). In this MIDI elaboration, the three bits of the velocity byte are reassigned for enharmonic representation. Our reasoning was that a span of 128 integers to represent the full spectrum of dynamic information (which is often blank, set universally at 64, or modulated to contrast piano and forte indications at some arbitrary distance from 0, 64, and 127), contained a lot of unused bits which, translated into actual MIDI performance, fell below the threshold of human hearing.

Current and Future Uses

The base-40 representation has been used in a host of software including programs for music theory (*MacGamut*), music analysis (*Humdrum* tools), music query (transport-distance project-in-progress at the University of Utrecht), and the visualization of tonal music (research-in-progress by Craig Stuart Sapp; see www://keyscapes.sapp.org.

Although *MuseData* does not *per se* incorporate base-40 integers in its representation, it is reliably converted to base-40 because ever pitch is fully identified with respect to its three parameters. Any system which is similarly explicit in its description of every pitch provides a firm basis for the further development of notation, analytical, and pedagogical applications.

Given the current focus of MusicNetwork on XML representations, we point out that MusicXML has been heavily influenced by the *MuseData* format and also supports the explicit representation of every pitch. In a sense it provides an alternative to MIDIPlus, which has not to our knowledge been implemented in any commercial programs which focus primarily on notation. It is a viable for such applications, provided that the MIDIPlus files are created from symbolic data which provides specific pitch information. Any XML scheme for music representation will face the same choices, opportunities, and limitations. It is not possible to convert from nothing (in this case no enharmonic information) to something. Our firm view is that explicit enharmonic information is essential to robust notation software and the viability of pedagogical programs in music theory and general literacy.

References

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