Base-40 Arithmetic

Implications for Notation-Based Applications

Overview

- Base-40 definition

- Base-40 and MIDI I/O

- Base-40 and Notation

- Base-40 and MIDIPlus file format

- Base-40 and musical-data interchange

What is Base-40?

- A scheme for pitch representation suited to
  - Music which is tonal
  - Notation which is conventional
  - Repertories widely referenced in professional environments including
    - Research
    - Teaching
    - Performance
    - Publishing

Where did Base-40 come from?

- Conceived by Walter Hewlett (1986)
- Used extensively at CCARH in data operations (not in encoding per se)
- Explanation published in *Musikometrika* (1992)
- Reproduced at http://www.ccarh.org/publications/reprints/
- Further elaborated in U.S. Patent 5,675,100 (7 October 1997)

Why Base-40?

- Musical literacy
- Tonal legibility (common practice era)
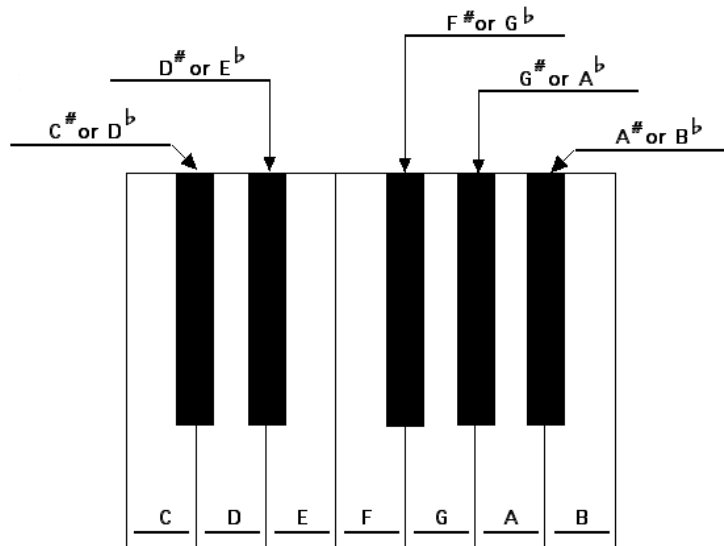- Musical computation in integer arithmetic
- Intervallic complementarity

> Base-10 complementarity:
>
> If interval = 3, complement = 7
>
> If interval = 6, complement = 4

Enharmonic-notation tiers

Physical instrument

A B C D E F G

C#or Db · D#or Eb · F#or Gb · G#or Ab · A#or Bb
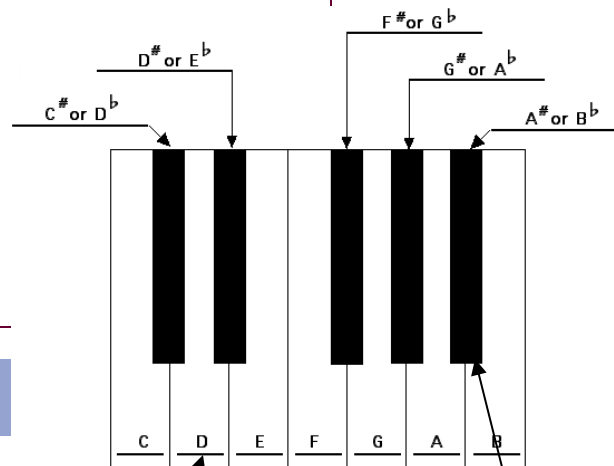
C D E F G A B

Cultural apparatus

- Letter names
  - Base-7 (0 #s/bs)
- Octave numbers
  - Base-12 (1#/b)
- Inflection names
  - Bases > 12 (>1#/b)

Enharmonic-notation tiers, cont.

Third tier
- ##
- #
- -
- b
- bb

(7 x 5) + 5

Fourth tier
- ###
- ##
- #
- -
- b
- bb
- bbb

C# or D♭   D# or E♭   F# or G♭   G# or A♭   A# or B♭

C   D   E   F   G   A   B

©EnchantedLearning.com

C## / D / Ebb

D## / E / Fb

A# / Bb / Cbb

MIDI I/0 vs. Graphical Notation

- MIDI to base-7

- MIDI to base-12

- MIDI to base-21

- MIDI to base-40

Solution: Translate from symbolic code to MIDIPlus

What is MIDIPlus?

- A binary implementation of base-40
- Replaces last 3 bits of velocity byte
- Used to interpret key number

MIDIPLUS Correlation of Pitch Spelling to Specific MIDI Velocity Values

Value	Notated Pitch										
89	D♭♭	D♭	E♭♭	F♭♭	F♭	G♭♭	G♭	A♭♭	A♭	B♭♭	C♭♭	C♭
90	C	C♯	D	E♭	E	F	F♯	G	G♯	A	B♭	B
91	B♯	B♯♯	C♯♯	D♯	D♯♯	E♯	E♯♯	F♯♯	F♯♯♯	G♯♯	A♯	A♯♯

MIDIPlus in Printing

- Raw MIDI to Notation (Bach Prelude in E Minor, BWV 855

BWV855 RawMIDI

J. S. Bach WTC-I Fugue 10

| BASE-40 | 175 | 186 | 198 | 215 | **214** | 215 | 209 | 215 | **208** | 215 | 203 | 215 | 198 | 215 | **214** | 215 | **197** | **208** | 186 | 181 | 186 | **197** | 181 | 175 | 169 | 198 |
| MIDIPLUS | 90 | 90 | 90 | 90 | **90** | 90 | 90 | 90 | **89** | 90 | 90 | 90 | 90 | 90 | **90** | 90 | **90** | **89** | 90 | 90 | 90 | **90** | 90 | 90 | 90 | 90 |

- Translation from symbolic code (*MuseData*) to *MIDIPlus* to notation

BWV855 With Correct Spellings

J. S. Bach WTC-I Fugue 10

| BASE-40 | 175 | 186 | 198 | 215 | **210** | 215 | 209 | 215 | **204** | 215 | 203 | 215 | 198 | 215 | **210** | 215 | **193** | **204** | 186 | 181 | 186 | **193** | 181 | 175 | 169 | 198 |
| MIDIPLUS | 90 | 90 | 90 | 90 | **91** | 90 | 90 | 90 | **90** | 90 | 90 | 90 | 90 | 90 | **91** | 90 | **90** | **90** | 90 | 90 | 90 | **91** | 90 | 90 | 90 | 90 |

Base-40 in Music Theory

- Scales

- Intervals

Chords (interval complementarity)

- ### Intervallic complementarity

| m2 + M7 = 8ve | m3 + M6 = 8ve | dim4 + aug5 = 8ve | dim5 + aug4 = 8ve | m6 + M3 = 8ve | m7 + M2 = 8ve |

| aug2 + dim7 | aug3 + dim6 | aug4 + dim5 | aug5 + dim4 | aug6 + dim3 |

- ### Chord definitions

Major   Minor   Augmented   Diminished

Summary

- Enharmonic details matter!
- Where?
    - Music notation
    - Music theory
    - Cultural discussions of tonal music
- When?
    - In symbolic code
    - In graphical output