



# **Harmonic Models (2)**



CS 275B/Music 254

# Harmonic Models: Overview

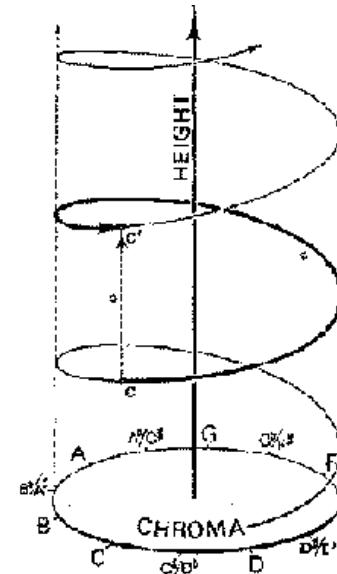
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- ▶ Geometric models
  - ▶ 18<sup>th</sup>-century Germany
    - ▶ Henichen
    - ▶ Euler
  - ▶ 19<sup>th</sup>-century Germany
    - ▶ Riemann
  - ▶ Krumhansl (1990)
  - ▶ Purwins (2000-2006)
  - ▶ Chew (2000-2006)
- ▶ Acoustic models
- ▶ Metric and spectral models
- ▶ Harmony foundation as basis for composition

# Geometric Models

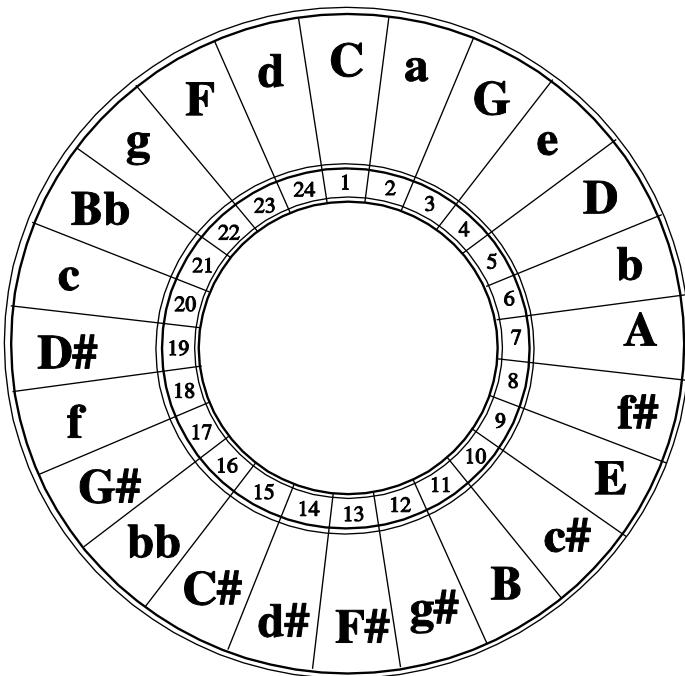
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Heinichen's Circle of Fifths (1728)

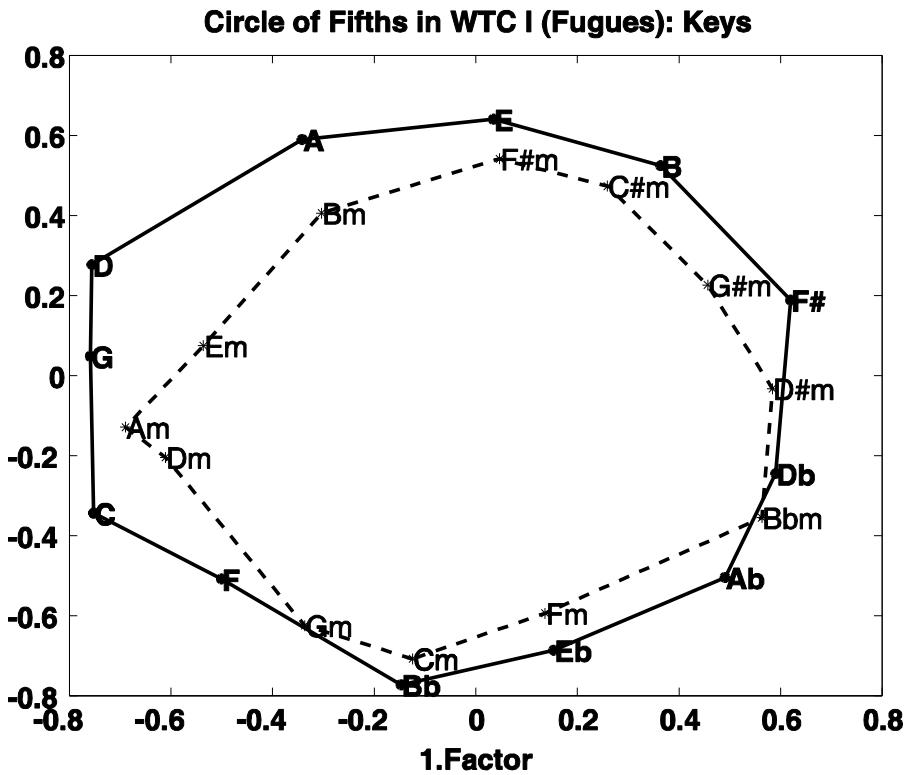


Chew's spiral array (2000)

# Acoustical properties of the Circle of Fifths



Modern arrangement of  
Circle of Fifths



Ozgur Izmirli (CT), Computing in Musicology 15  
(2008): differential utilization of key regions in WTC  
fugues

# Toroidal model of tonality

Hendrik Purwins, Technische Univ., Berlin, (c. 2000)

Weber (18<sup>th</sup> cent) key chart

d♯ F♯ f♯ A a C c

g♯ B b D d F f

c♯ E e G g B♭ b♭

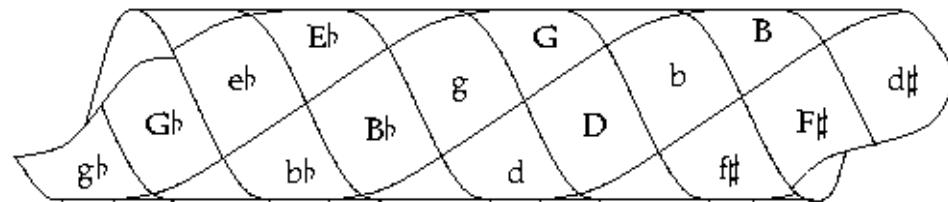
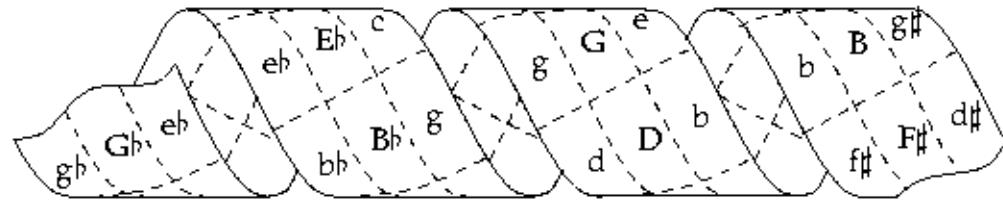
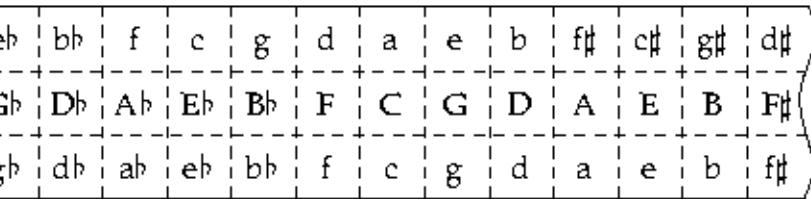
f♯ A a C c E♭ e♭

b D d F f A♭ a♭

e G g B♭ b♭ D♭ d♭

a C c E♭ e♭ G♭ g♭

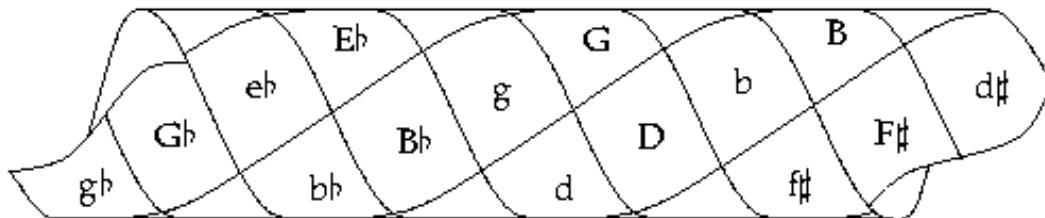
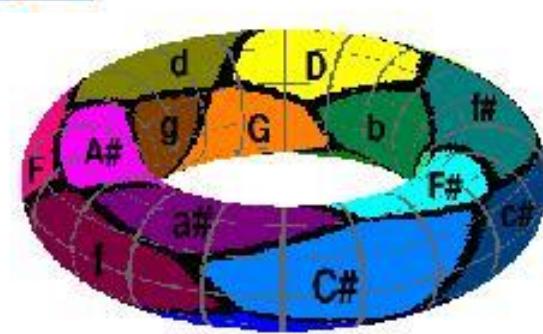
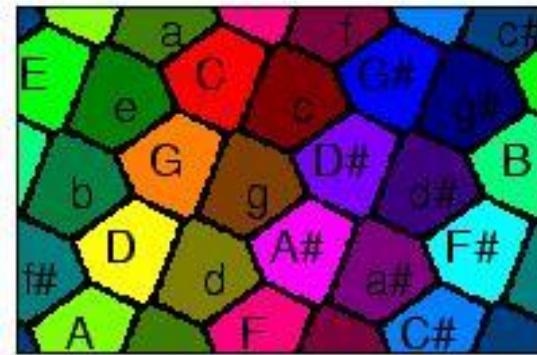
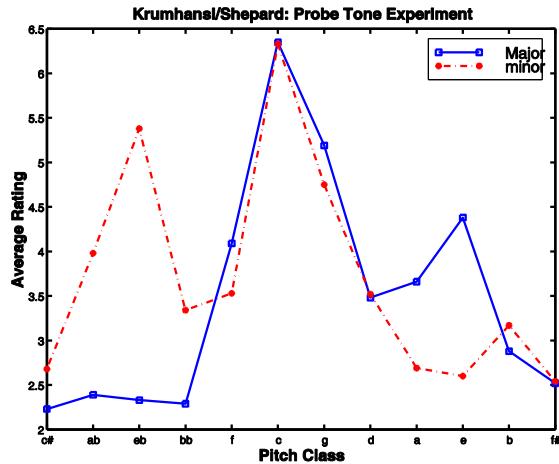
e♭	b♭	f	c	g	d	a	e	b	f♯	c♯	g♯	d♯
G♭	D♭	A♭	E♭	B♭	F	C	G	D	A	E	B	F♯
g♭	d♭	a♭	e♭	b♭	f	c	g	d	a	e	b	f♯



Derived from transformation of  
18<sup>th</sup>-century German grids to  
three-dimensional mappings

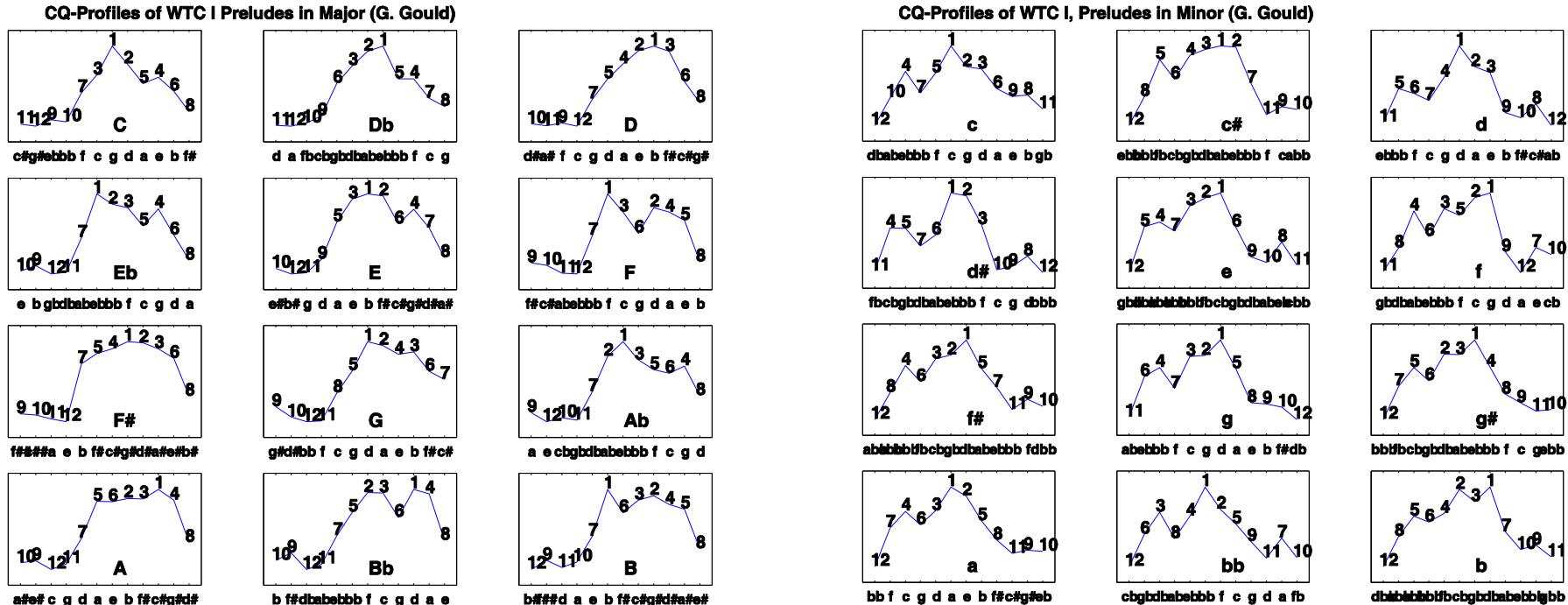
# Krumhansl: *Cognitive Foundations of Musical Pitch* (1990)

Probe-tone experiment results: How well do subjects judge how well a single tone  
Relates to a musical passage? Intercultural studies show that judgments vary by mode (major/minor).



# Well Tempered Clavier (Purwins)

## Key frequency in Gould performances of Bach's WTC



# Spectral Weights (in association with metric weights)

The image shows two staves of musical notation in 3/4 time, treble and bass clef, with a key signature of three flats. Vertical lines connect specific notes in each measure, representing spectral weights. Below the music, a red horizontal bar spans the width of the measures. Underneath the bar, a series of black stars are placed below the notes, indicating metric weights. The first measure starts with a vertical line above the first note, followed by a star. The second measure has a vertical line above the first note, followed by three stars. The third measure has a vertical line above the first note, followed by three stars. The fourth measure has a vertical line above the first note, followed by four stars. The fifth measure has a vertical line above the first note, followed by five stars. The sixth measure has a vertical line above the first note, followed by five stars. The seventh measure has a vertical line above the first note, followed by five stars. The eighth measure has a vertical line above the first note, followed by five stars.

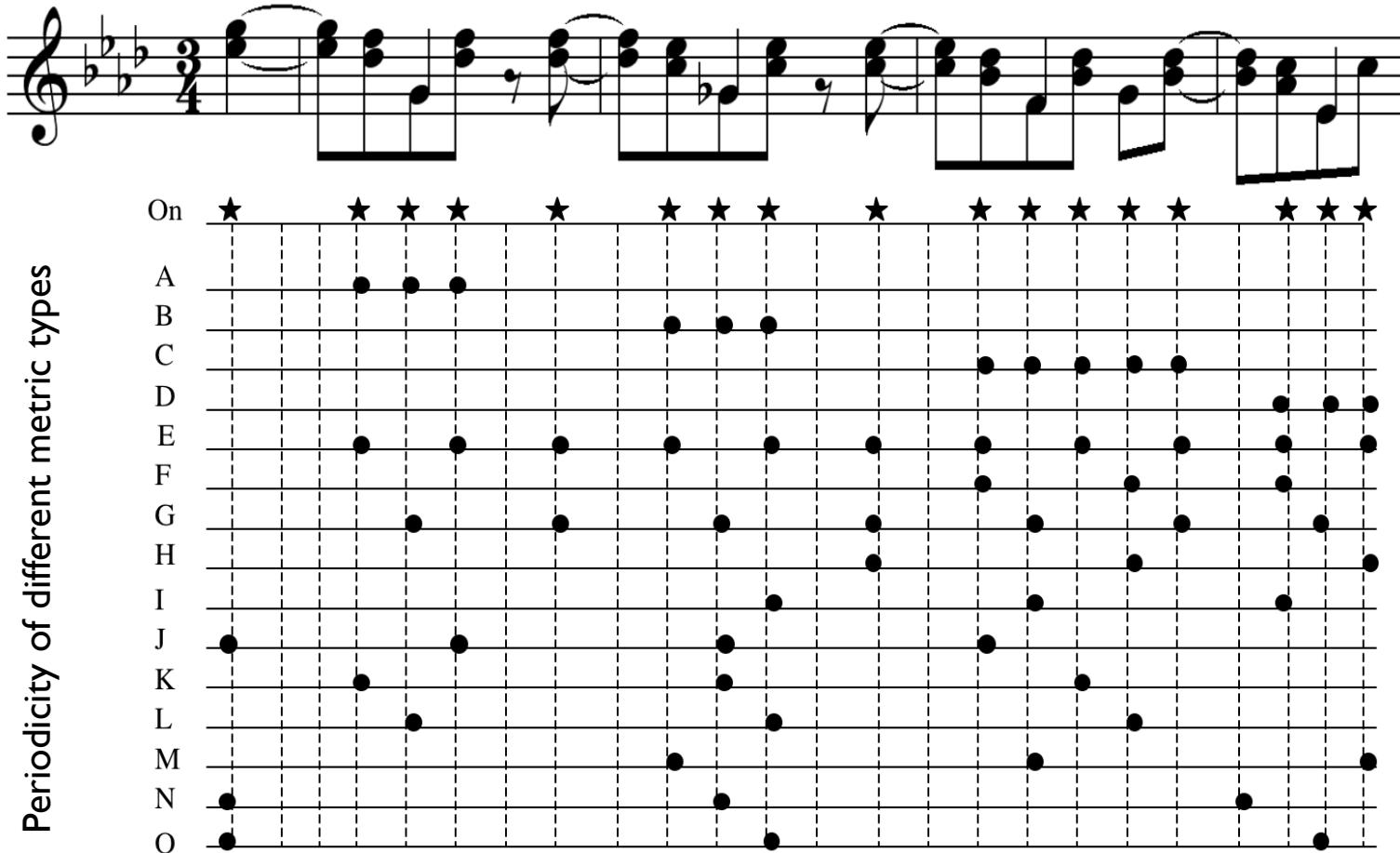
On ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★ ★

Schumann Waltz Op. 124, N. 15, mm. 1-4

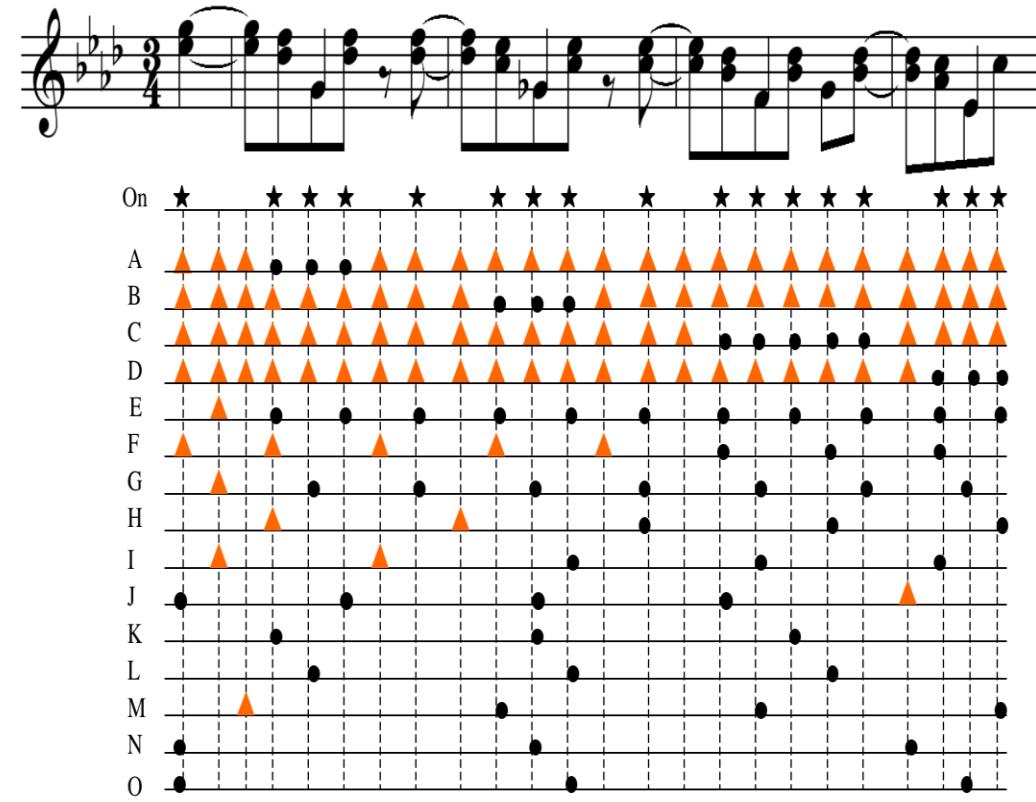
Work of Anja Volk (Utrecht Univ.)

- **inner metric structure** [outer metric structure = meter]

# Volk's Inner metric structure



# Inner vs outer metric structure

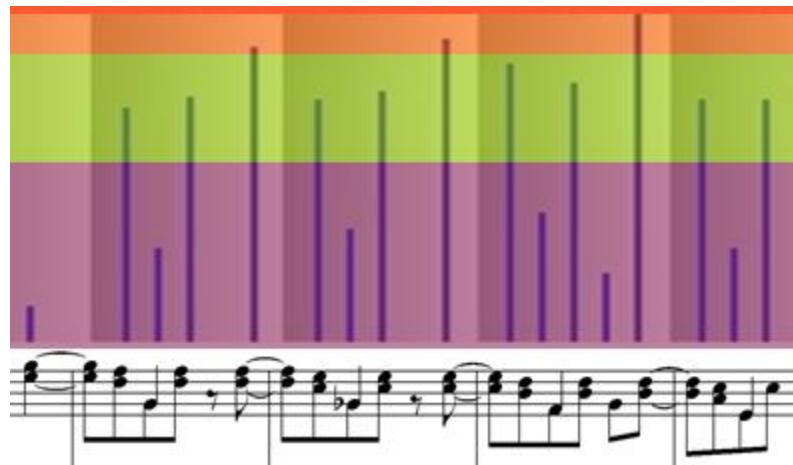


# Spectral weights

$$m_{s,d,k} = \{s + id, i = 0, \dots, k\}$$

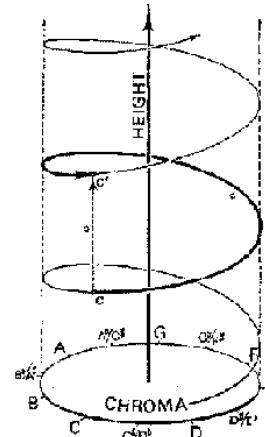
~~start~~      ~~period~~      ~~length~~

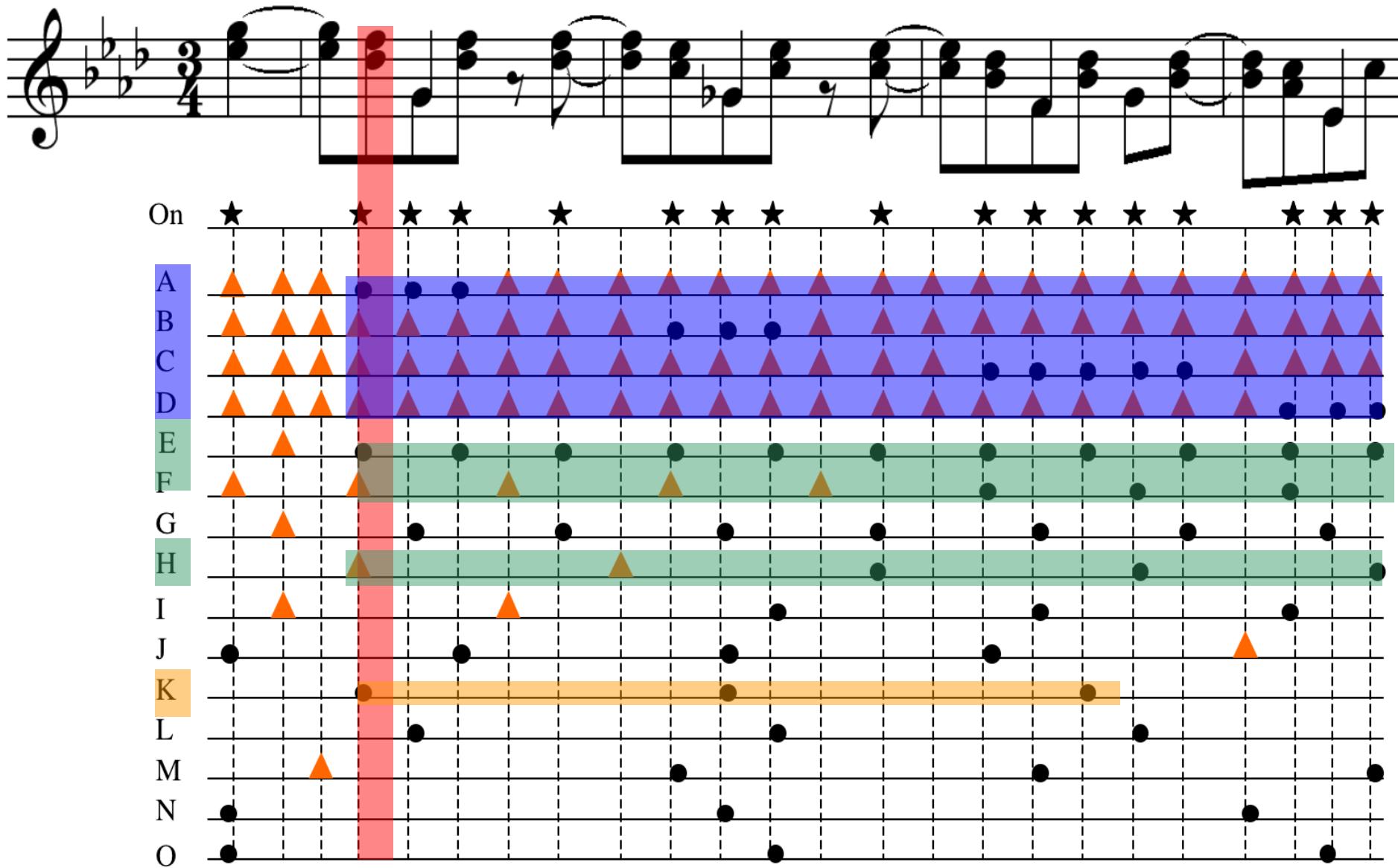
$$W_{l,p}(o) = \sum_{\forall m_k : k \geq l, o \in m_k} k^p$$



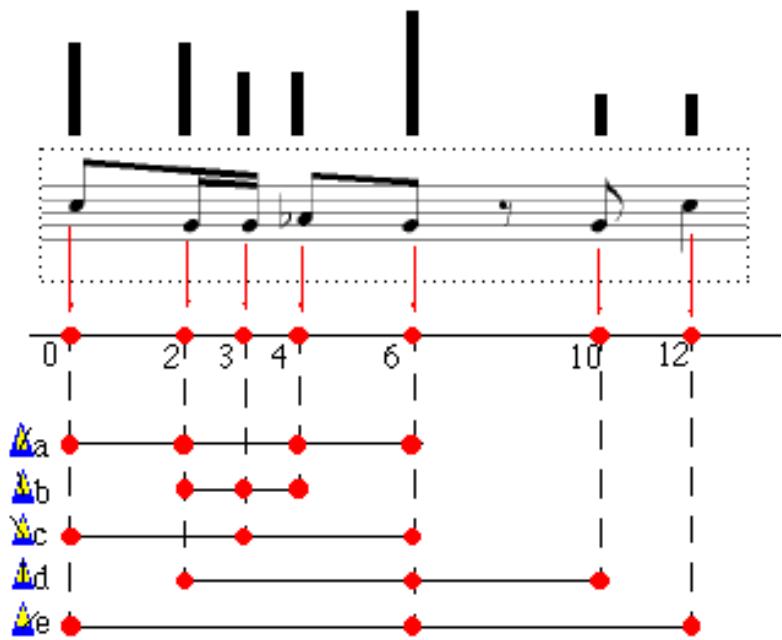
Work of Anja Volk (Utrecht Univ.)

- inner metric structure [outer metric structure = meter]
- **Spectral weights** (additive from inner metric weight grid)





## Inner metric structure (summary)



Author's summary of process

Prozess

-Ausgangspunkt ist midi-Repräsentation d.Stückes, am Ende erhält man für jede Note ein metrisches Gewicht, dass die metr. Bedeutung dieses Tones kodieren soll

-metr. Analyse berücksichtigt nun lediglich die Einsatzzeiten der Töne, vergißt Tonhöhe und Dauern

-sucht nach Regularitäten in den EZ, indem nach wiederholenden Einsatzzeitenabständen gesucht wird

-d.h. ermittelt werden die sog. Lokalen Metren, dies sind Raster oder Kämme von Tönen im gleichen Einsatzzeitenabstand

-metr. Gewicht einer EZ berechnet sich als gewichtete Summe aus allen lokalen Metren, an denen sie beteiligt ist (man „mißt“ die Regularitäten)

-höchstes metr. Gewicht auf G entspricht nicht unserer Erwartung:

-Frage: was kann man denn erwarten von diesem Ansatz?

# *Partimenti:* Harmonic foundation as basis of composition

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- ▶ 18<sup>th</sup> Neapolitan practice of composition
- ▶ Sanguinetti (2012): *The Part of Partimento*
- ▶ Robert Gjerdingen (2010): <http://faculty-web.at.northwestern.edu/music/gjerdingen/partimenti/collections/Durante/diminuti/index.htm>
- ▶

# Gjerdingen: Partimenti website

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| The Brothers Greco (1650s–1720s)  | Leonardo Leo (1694–1744)        |
| Stanislao Mattei (1750–1825)      | Giovanni Paisiello (1741–1816)  |
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Northwestern University

# Sanguinetti: Partimento book

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- ▶ The Rule of the Octave
- ▶ Suspensions
- ▶ Elaborations of the Rule of the Octave
- ▶ Relationships to compositional genres
- ▶ How to create your own partimenti

*The Art of Partimento* (OUP, 2012)

# Sanguinetti, 2

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- ▶ **The Rule of the Octave**
  - ▶ How to harmonize an octave
    - ▶ Ascending/descending
    - ▶ Major/minor
  - ▶ Suspensions
  - ▶ Elaborations of the Rule of the Octave
  - ▶ Relationships to compositional genres
  - ▶ How to create your own partimenti

# Sanguinetti, 3

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- ▶ The Rule of the Octave
  - ▶ How to harmonize an octave
    - ▶ Ascending/descending
    - ▶ Major/minor
- ▶ Suspensions
  - ▶ In the soprano: by the 4<sup>th</sup>, by the 7<sup>th</sup>, by the 9<sup>th</sup>
  - ▶ In the bass: by the 2nd
- ▶ Elaborations of the Rule of the Octave
- ▶ Relationships to compositional genres
- ▶ How to create your own *partimenti*

# Sanguinetti, 4

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- ▶ **The Rule of the Octave**
  - ▶ How to harmonize an octave
    - ▶ Ascending/descending
    - ▶ Major/minor
- ▶ **Suspensions**
  - ▶ In the soprano: by the 4<sup>th</sup>, by the 7<sup>th</sup>, by the 9<sup>th</sup>
  - ▶ In the bass: by the 2nd
- ▶ **Elaborations of the Rule of the Octave**
  - ▶ Interpolations in the bass line
  - ▶ Variations, diminutions in any part
  - ▶ Motives, subjects et al.
- ▶ Relationships to compositional genres
- ▶ How to create your own *partimenti*

# Examples: Ascending, descending scales

- ▶ Octave, ascending, 3 positions

The image shows three staves of musical notation for a treble clef instrument. Each staff has a key signature of one sharp (F#) and a common time signature. The first staff is labeled "First position". The second staff is labeled "Second position". The third staff is labeled "Third position". Each staff consists of eight measures of music. Below each measure, there is a numerical sequence representing the fingerings used to play the notes. For example, in the first staff, the fingerings are: 8, 5, 3; 6, 4, 3; 8, 6; 6, 5; 8, 6; 6, 3; 8, 6; 8. The fingerings follow a repeating pattern of two pairs of numbers per measure.

- ▶ Ascending, minor, 1<sup>st</sup> position

The image shows a single staff of musical notation for a treble clef instrument. The staff has a key signature of one flat (B-flat) and a common time signature. It is labeled "First position". The staff consists of eight measures of music. Below each measure, there is a numerical sequence representing the fingerings used to play the notes. The fingerings are: 8, 5, 3; 6, 4, 3; 8, 6; 6, 5; 8, 6; 6, 3; 8, 6; 8. The fingerings follow a repeating pattern of two pairs of numbers per measure.

- Descending, major, 1<sup>st</sup> position

The image shows a single staff of musical notation for a treble clef instrument. The staff has a key signature of one sharp (F#) and a common time signature. It is labeled "First position". The staff consists of eight measures of music. Below each measure, there is a numerical sequence representing the fingerings used to play the notes. The fingerings are: 8, 5, 3; 6, 4, 3; 8, 6; 6, 5; 8, 6; 6, 3; 8, 6; 8. The fingerings follow a repeating pattern of two pairs of numbers per measure.

## Positions:

Treble starting on the first, 3<sup>rd</sup>, 5th

# Examples: Suspensions

## ▶ Fourth suspensions

a) Treble and bass staves in G major (C key signature). Measure 1: Bass note 8, treble note 4. Measure 2: Bass note 8, treble note 5. Measure 3: Bass note 8, treble note 3.

## ▶ Seventh suspensions

a) Treble and bass staves in G major (C key signature). Measure 1: Bass note 8, treble note 7. Measure 2: Bass note 8, treble note 6. Measure 3: Bass note 8, treble note 5.

4<sup>th</sup>, 7<sup>th</sup>, 9<sup>th</sup> suspensions  
pertain to treble

## ▶ Ninth suspensions

c) Treble and bass staves in G major (C key signature). Measure 1: Bass note 8, treble note 3. Measure 2: Bass note 8, treble note 9. Measure 3: Bass note 8, treble note 6.

As in exachordal theory, a “mutated” bass thrusts the harmonic action into a new key.

- Mutated bass
- “Major,” “minor” fourths

Bach, WTC I, Prelude 1 (bb. 1-11)

c) Treble and bass staves. The bass line shows two types of fourths: “minor” fourth (e.g., bass note 8 followed by 5) and “major” fourth (e.g., bass note 8 followed by 2). An arrow points to the transition between these two types, labeled “scale mutation.”

# Examples: Rhythmic variation

- ▶ Limping suspensions

a)

Fenaroli

Musical score for 'Limping suspensions' (a). It consists of two staves. The top staff is in treble clef and common time, with a key signature of one sharp. The bottom staff is in bass clef and common time, with a key signature of one sharp. The music features eighth-note patterns with various rhythmic values above them, such as '5' and '6'. The score is labeled 'Fenaroli'.

- ▶ Rhythmic enrichment

c)

Valente

Musical score for 'Rhythmic enrichment' (c). It consists of two staves. The top staff is in treble clef and common time, with a key signature of one sharp. The bottom staff is in bass clef and common time, with a key signature of one sharp. The music features eighth-note patterns with various rhythmic values above them, such as '9', '8', 'x', '5', '3', '7', '6', '4', and '2'. The score is labeled 'Valente'.

# Examples: Patterned basses

## ► Sequential bass

a)

model      1st transposition      2nd transposition

## ► Patterns based elaboration

- Rising by 3rds, falling by step
- Falling by 4ths, rising by step

b)

6 4 3 6 4 3 6 4 3 6 5 4 5 #

c)

8 4 6 5 9 4 8 3 6 4 5 9 4 8 6 5 5 4 5 4 5