Base-40 arithmetic for music apps

Music 253/CS 275A

Stanford University

Where did Base-40 come from?

- Conceived by Walter Hewlett (1986); first pub 1992
- *Goals:* enharmonic spelling preservation, correct analysis, correct transposition
- Reproduced at http://www.ccarh.org/publications/reprints/
- Further elaborated in U.S. Patent 5,675,100 (7 October 1997) http://www.google.com/patents/US5675100

Common bases in musical arithmetic

Subdivisions of the octave

- Base 7 (diatonic)
- Base 12 (semi-chromatic; MIDI)—favors eq-temp **sound**
- Base 21 (fully chromatic through 1 #/b)—favors simple **notation**
- ????? (19, 35….)
- Base 40 (fully chromatic through 2 #/b; supports invertible intervals for **analysis**)

Why Base-40?

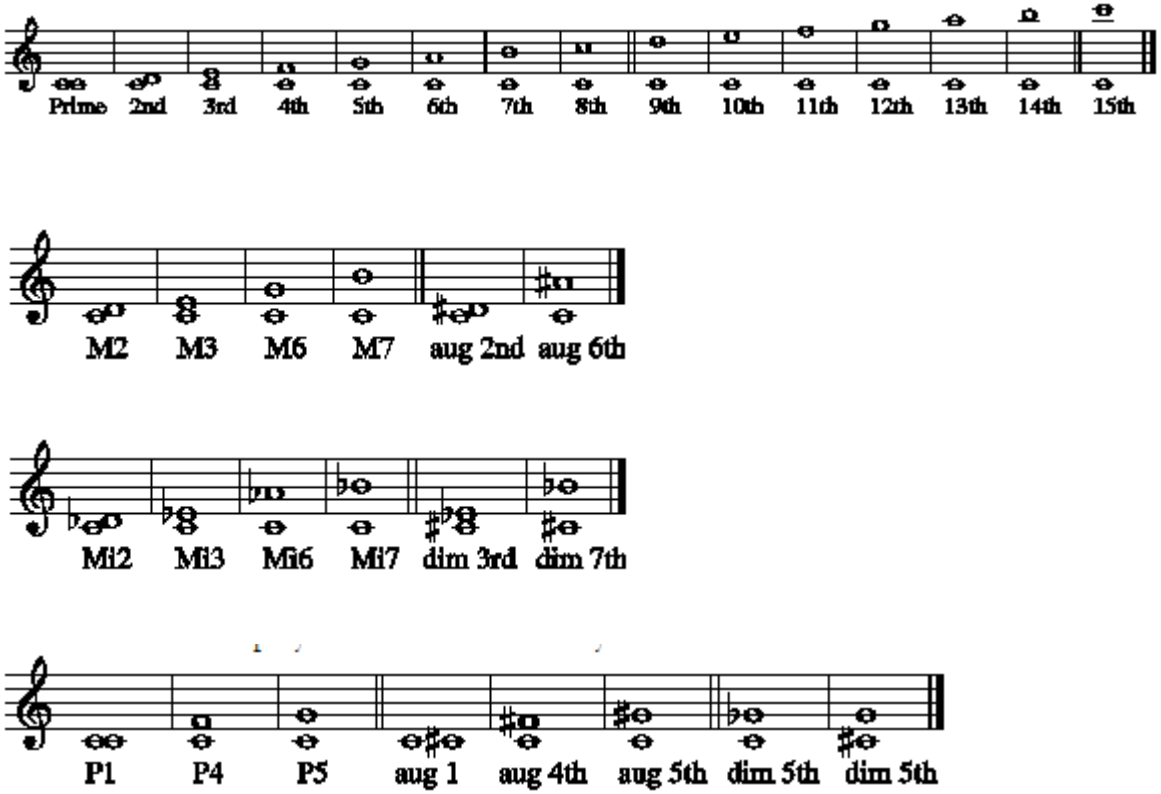
- Musical literacy
- Tonal legibility (common-practice era)
- Musical computation in **integer arithmetic**
- Intervallic **complementarity**

Base-10 **complementarity**:

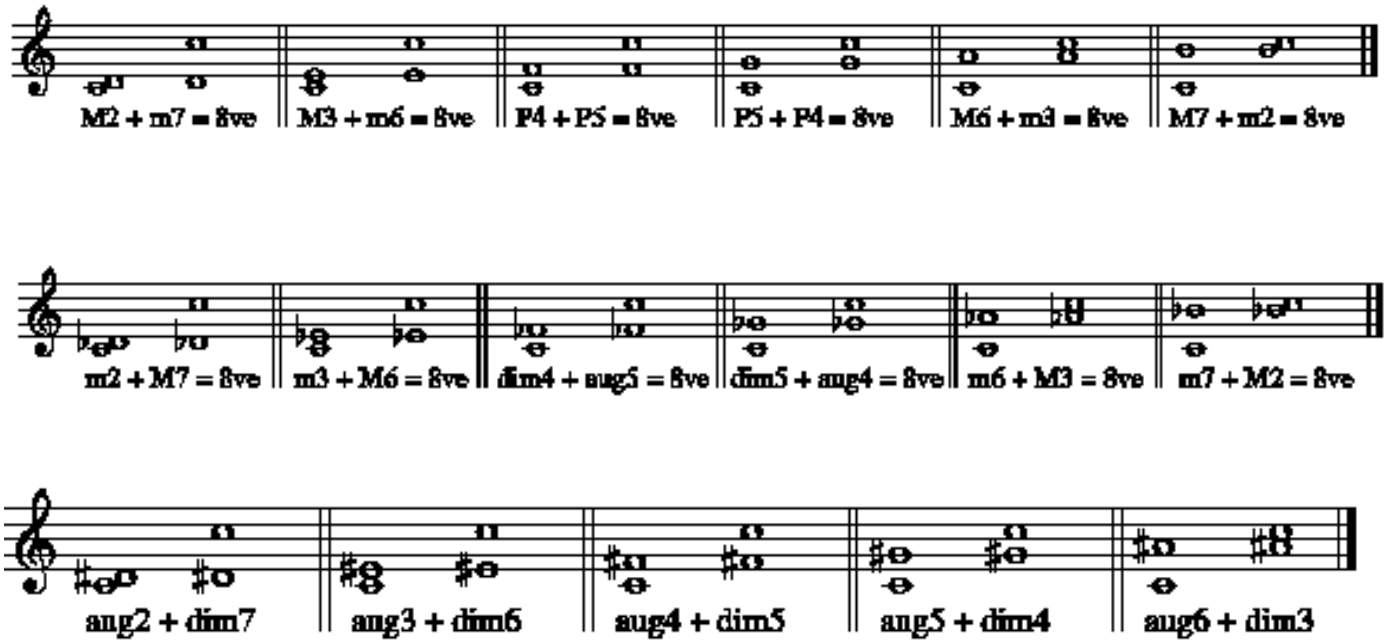
If interval = 3, complement = 7

If interval = 6, complement = 4

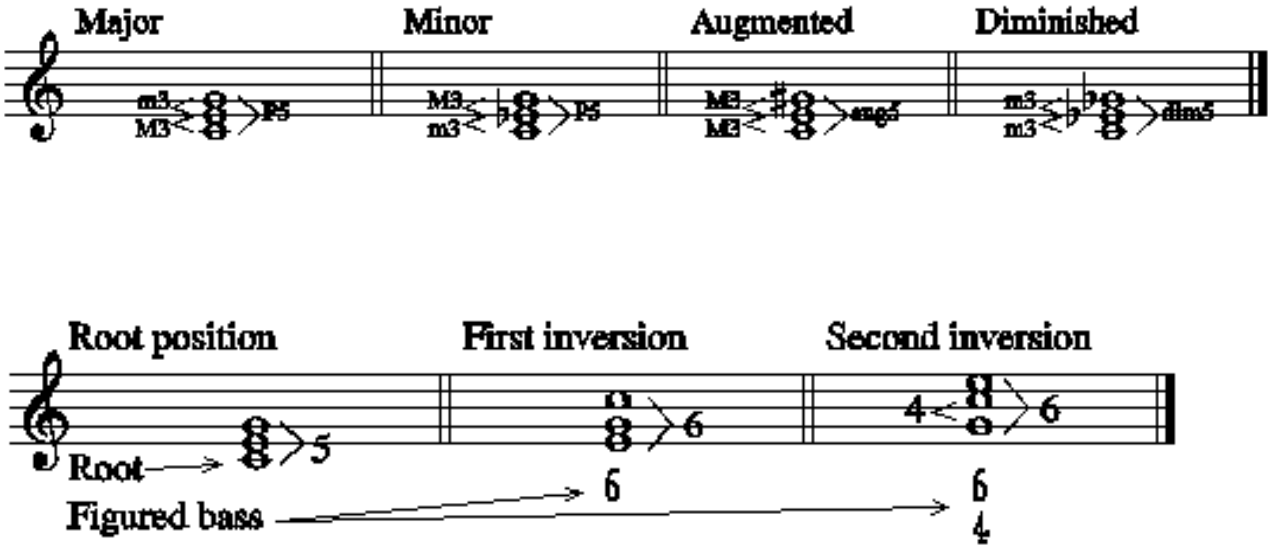
Review: Interval sizes and qualities



Review: Intervallic complementarity



Review: Intervallic complementarity in chords

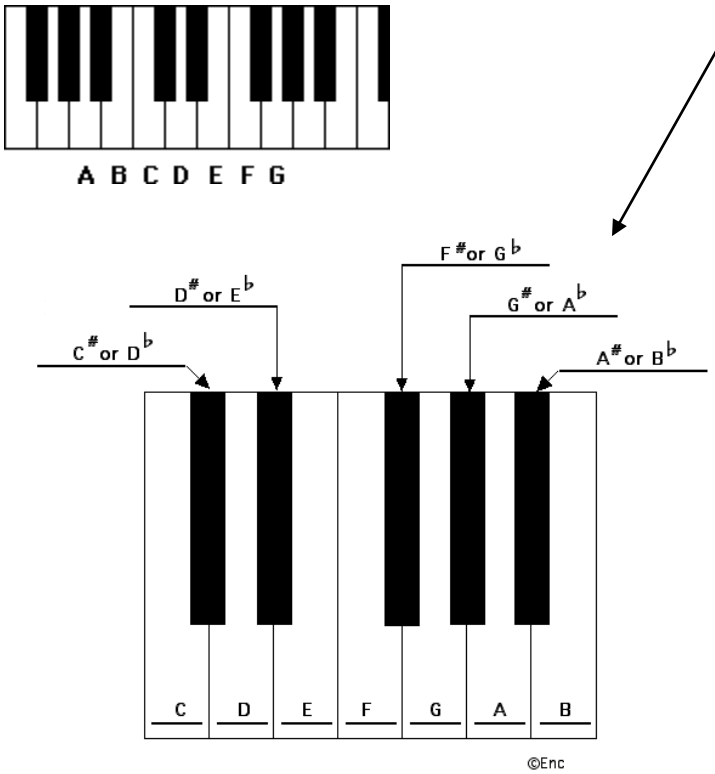


Integer arithmetic in digital analysis

- *Binomial solutions*: Brinkman, Böker-Heil
  - Required 3 params (pitch name, octave number, inflection)
- *Arbitrary mappings*: C=10, D=20, E=30….
  - Same-sized intervals do not always produced same numbers (depends on endpoints: F-E = 10, Eb-D = 9)
- Base-40 is **interval-invariant**:
  - it produces consistent arithmetical results
  - irrespective of endpoints and without binomials

Enharmonic-notation tiers

- Physical instrument



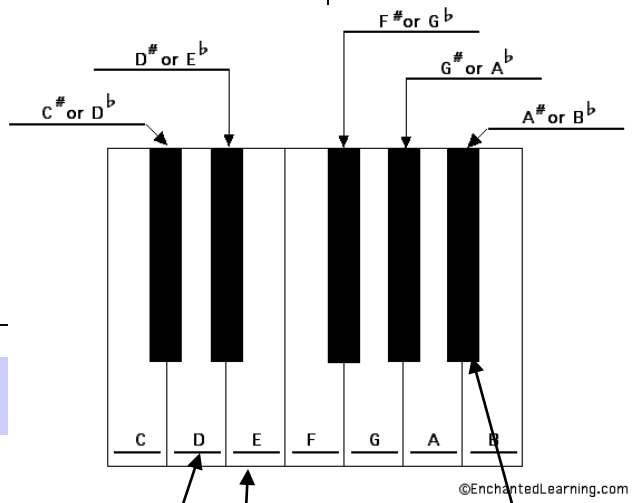
- Cultural apparatus
  - *Letter names*
    - Base-7 (0 #s/bs)
  - *Octave numbers*
    - Base-12 (1#/b)
  - *Inflection names*
    - Base-21 (1#/b)
  - Inflection names
    - Base-40 (2#/b)

Wider system: Enharmonic-notation tiers

- Third tier
  - ##
  - #
  - -
  - b
  - bb

- Fourth tier
  - ###
  - ##
  - #
  - -
  - b
  - bb
  - bbb

(7 x 5) + 5



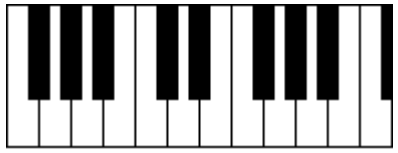
C## / D / Ebb

D## / E / Fb

A# / Bb / Cbb

Base-40 Rule

Simple rule: Where a whole step exists between two key names,
a null token is used.



Representation		
C𝄫1	=	1
C𝄫♯1	=	2
C𝄪1	=	3
C♯1	=	4
C♯♯1	=	5
.	=	6
D𝄫1	=	7
D𝄫♯1	=	8
D1	=	9
D♯1	=	10
D♯♯1	=	11
.	=	12
E𝄫1	=	13
E𝄫♯1	=	14
E1	=	15
E♯1	=	16
E♯♯1	=	17
F𝄫1	=	18
F𝄫♯1	=	19
F1	=	20
F♯1	=	21
F♯♯1	=	22
.	=	23
G𝄫1	=	24
G𝄫♯1	=	25
G1	=	26
G♯1	=	27
G♯♯1	=	28
.	=	29
A𝄫1	=	30
A𝄫♯1	=	31
A1	=	32
A♯1	=	33
A♯♯1	=	34
.	=	35
B𝄫1	=	36
B𝄫♯1	=	37
B1	=	38
B♯1	=	39
B♯♯1	=	40

Computation of Intervals

Interval	Delta	Interval	Delta
perfect unison	0	perfect octave	40
aug. unison	1	dim. octave	39
dim. second	4	aug. seventh	36
minor second	5	major seventh	35
major second	6	minor seventh	34
aug. second	7	dim. seventh	33
dim. third	10	aug. sixth	30
minor third	11	major sixth	29
major third	12	minor sixth	28
aug. third	13	dim. sixth	27
dim. fourth	16	aug. fifth	24
perfect fourth	17	perfect fifth	23
aug. fourth	18	dim.fifth	22

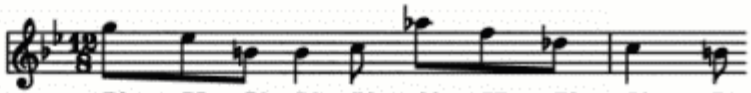
2. The inversion of a simple interval is forty minus that interval.

3. Intervals may be computed across the B - C octave boundary without extra calculations.

4. Compound intervals such as tenths are related to intervals by the difference of an octave (40). A major tenth is 12 + 40 = 52.

5. Limitations: Intervals involving not… set, e.g. with three or more sharps o… cannot be computed properly from this … sentation. Some unusual intervals wi… which overlap the numbers for the sta… intervals given above. For example, … quadruple augmented unison between C… C♯♯1 has an interval value of 6, whic… the number for a diminished second. … limitations can be removed by using a… of a higher order.

Example 4: *"Seufzer, Tränen, Kummer, Not"* from Cantata 21, *Ich hatte viel Bekümmer*



MIDI representation:	79	75	71	71	72	80	77	73	72	71
MIDI interval size:	4	4	0	1	8	3	4	1	1	
Base-40 representation:	186	174	158	158	163	191	180	168	163	158
Base-40 interval size:	12	16	0	5	28	11	12	5	5	
Standard interval name:	M3	d4	-	m2	m6	m3	M3	m2	m2	

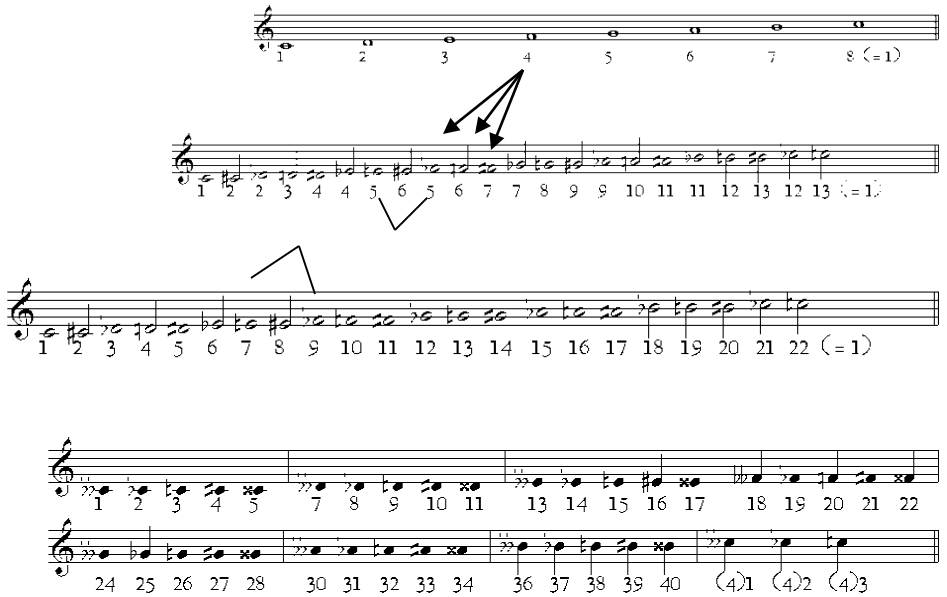
From Base-40 to enharmonic preservation

MIDI to base-7

MIDI to base-12

MIDI to base-21

MIDI to base-40



Solution: Translate from symbolic code to MIDIPlus

What is MIDIPlus?

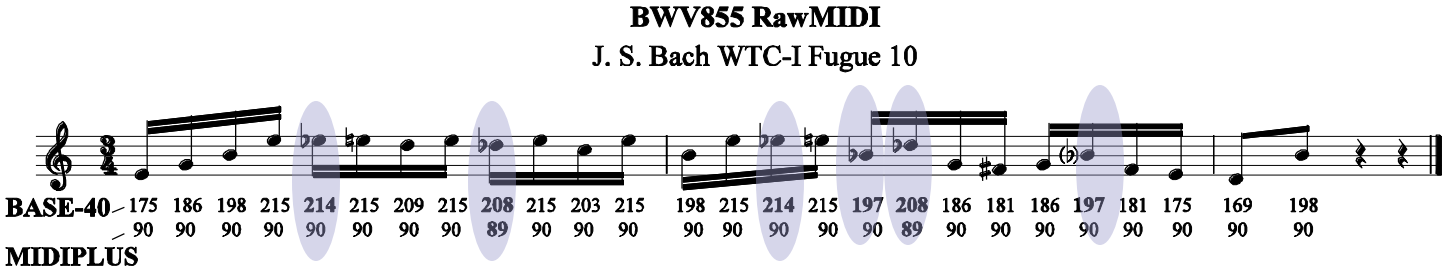
- In MIDI file format, a binary implementation of base-40
- Replaces last 3 bits of velocity byte
- Used to interpret key number

MIDIPLUS Correlation of Pitch Spelling to Specific MIDI Velocity Values

Value	Notated Pitch											
89	D𝄫	D𝄫	E𝄫	F𝄫	F𝄫	G𝄫	G𝄫	A𝄫	A𝄫	B𝄫	C𝄫	C𝄫
90	C	C♯	D	E𝄫	E	F	F♯	G	G♯	A	B𝄫	B
91	B♯	B♯♯	C♯♯	D♯	D♯♯	E♯	E♯♯	F♯♯	F♯♯♯	G♯♯	A♯	A♯♯

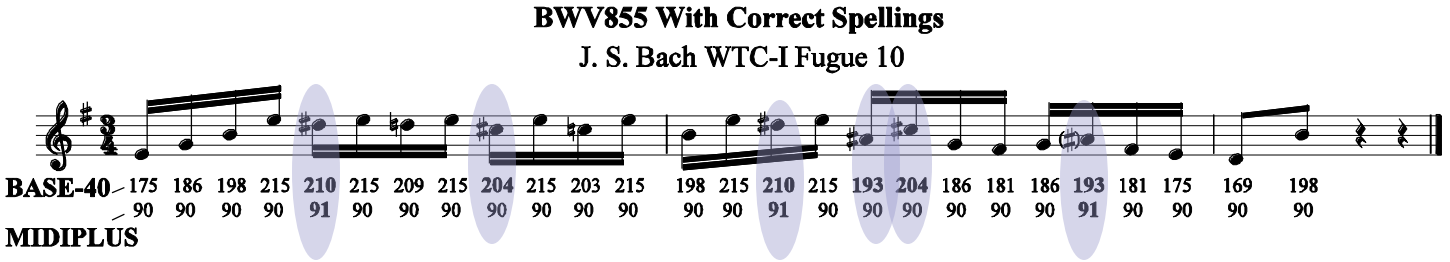
MIDIPlus in Printing

Raw MIDI to Notation (Bach Prelude in E Minor, BWV 855



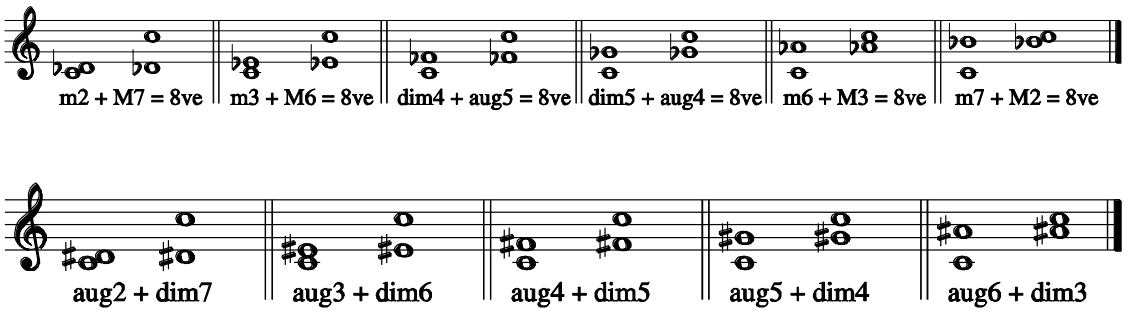
Translation from symbolic code (*MuseData*) to *MIDIPlus*

to notation



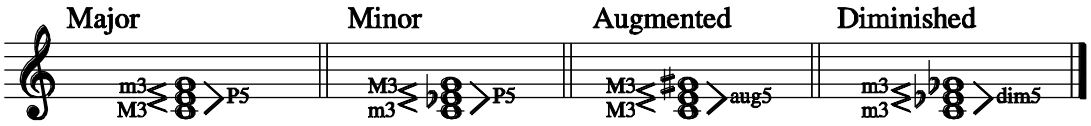
Chords (intervallic complementarity)

- Intervallic complementarity



3 9 15 20 26 32 38 43

- Chord definitions



Relevant handouts

Two translations of BWV 855 expressed with base-40

- E-Minor Fugue with enharmonically correct notation
  - http://esf.ccarh.org/MusicTheory_Tutorials/Base40_Handout_supp1.PDF
- E-Minor Fugue via MIDI-to-notation:
  - http://esf.ccarh.org/MusicTheory_Tutorials/Base40_Handout_supp2.PDF

Music theory tutorial:
http://esf.ccarh.org/MusicTheory_Tutorials/MusicTheory_ComputerApps.htm